TWO EVALUATIVE MODELS FOR A FAMILY OF SUBMARINE VERSUS SUBMARINE EXPANDING SQUARE SEARCH PLANS

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THESIS

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of
Submarine Versus Submarine
Expanding Square Search Plans

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ABSTRACT

This thesis investigates the effectiveness of a search plan developed by B. O. Koopman in a submarine versus submarine search situation. Two computer simulation models allow probability of target detection as a function of sonar range to be used as a measure of effectiveness. The Koopman search plan is analyzed and a family of alternate search plans are developed. The choice of a particular alternate search plan is dependent on the parameters of the problem. These parameters are target speed, searcher speed, time late to the search area and total time available to conduct the search. By use of the computer programs a search plan can be chosen so as to maximize the probability of target detection at a particular sonar range for each combination of input parameters.

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I. INTRODUCTION

During World War II a great deal of analytical and statistical work was done in the field of military operations. In particular the Navy benefited greatly from the work of the Operations Research Group (ORG) headed by Bernard O. Koopman. The Group's research has been compiled into a single volume entitled Search and Screening (OEG Report No. 56). This volume explores the application of the scientific method to tactical problems of naval warfare. It is intended to serve as a theoretical framework and foundation for more immediate practical studies. Philip M. Morse in the forward to Search and Screening (OEG Report No. 56) stated:

> "Although the tactical doctrines presented apply to instruments, weapons and conditions prevailing during World War II, it is believed that the methods and systematic processes of analysis which led to the doctrines have wide application--not only to submarine warfare but to many other military and civilian problems."

The analysis performed and the tactical applications devised by the Operations Research Group have withstood the test of time very well. Although extensive progress has been made in the methods of operations research since World War II, many of the tactical applications derived by the Operations Research Group are still considered the best available by current practitioners of the art.

In particular the methods developed by Dr. J.M. Dobbie for constructing expanding square searches for targets in transit have been widely applied. However, some of these applications may be inappropriate. One use they

have been applied to is that of a submerged submarine searching for another submerged submarine. In this particular situation the use of the search plan developed by Koopman and Dobbie is questionable. This thesis will investigate the appropriatness of the Koopman search plan in the submarine versus submarine search situation.

II. DISCUSSION OF THE KOOPMAN SEARCH PLAN

A. BASIC ASSUMPTIONS OF THE KOOPMAN SEARCH PLAN

In planning a search the nature of the target is usually known. The general position of the target is usually a random variable. Unless a fairly accurate estimate of its motion can be made, the search plan will have to be designed so as to be effective against a target having any one of many different sorts of motion. This plan assumes that the target speed is known exactly and that the target's course is a constant. When the object of search has had its approximate location disclosed at a certain time, the searcher has the problem of disposing its subsequent searching effort (which is always limited) in such a manner as to maximize its chance of detecting the target. This is subject of course to the searcher's limitations and the practical limitations of navigation.

It is assumed that information regarding the point of fix is received by the searcher. The time of this position information is also assumed known. It is further assumed that the searcher is airborne and has a considerable speed advantage over the target.

The information concerning the target's position is not exact. Only a probability distribution of target positions is actually given. This distribution will have its greatest density at the point of fix and fall continuously to zero at a distance. The distribution will be assumed to be circular normal in form. Therefore, the probability that the target at the time of fix will be in a certain small region dx dy at the point (x,y) a distance r from the origin (point of fix) is given by the following:

$$f(x,y) = f(r) = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \, dxdy$$

where σ is the standard deviation of the circular normal distribution.

It is assumed that the target's speed is known. The target's course is unknown but is assumed to be distributed uniformly between zero and 360 degrees. The target location distribution at a particular time, T hours after the initial fix at a distance r from the origin (point of fix), can be expressed by the following probability density:

$$f(x,y;T) = f(r;T) = \frac{1}{2\pi\sigma^2} e^{-(+r^2 + St^2T^2)/2\sigma^2)} \cdot I_0 \left(\frac{St\ rT}{2}\right)$$

The derivation of these relationships are shown in chapter two of Koopman [Ref. 1] where St is the estimated target speed. In this equation I_0 is the ordinary Besel function of zeroth order value for pure imaginary arguments of the function.

Figure 1 is a graph of $2\pi\sigma^2 f(r;T)$ this function plotted for various values of t as a function of r. It is obvious from figure one that when StT is greater than three sigma (3σ) the distribution has its maximum at r = StT.

Because of this a search plan can be constructed for large values of time which have elapsed from the time of fix to the time of initiation of the search. This time interval is called time late.

Theoretically the searcher should head for a position at a distance r where r = StT from the point of fix. This will place him on the peak

of the target location distribution which is circular in form. The search pattern should then be constructed so as to maintain the searcher's position at the point of maximum density of the target location distribution. This would result in a search plan that resembled an equiangular or logarithmic spiral. There are practical considerations which make this infeasible, namely navigation problems and the undesirability of constant helming. Therefore, a search plan was constructed which approximates the theoretically optimal plan.

B. CONSTRUCTION OF THE KOOPMAN SEARCH PLAN

The theoretically optimal search plan was approximated by an expanding square search plan. Each circuit or square consists of four legs with 90 degree turns. The shape of the squares are shown in figure 2. The leg lengths are L_1, L_2, L_3, etc., and the corresponding distances of the search legs from the point of fix are r_1, r_2, r_3, etc. The time required for the searcher to go from point A to point B is $(r_1 + r_2)/S_0$, where S_0 is the searcher's speed. The time required for the target to move from a distance r_1 from the point of fix to a distance r_2 from the point of fix is $(r_2 - r_1)/St$ where St is the target speed. The searcher will just keep up with the target if these times are equal. By this reasoning

$$r_2 = mr_1$$

$$r_3 = mr_2$$

$$r_4 = mr_3$$

where

$$m = \frac{S_0 + St}{S_0 - St}$$

If we were to make $r_5 = mr_4$, the fifth leg would duplicate the position of the first leg in space relative to the target. That is to say that the searcher would be on the highest density point of the target location distribution. However, having traveled the first circuit (four legs) so as to be at this point of maximum density and not having detected the target has imparted additional information to the searcher. The first circuit being unsuccessful has altered the target location distribution. The target location distribution after one circuit is no longer unimodal. It is now a bimodal distribution with a depression where the peak of the initial target distribution function had been. This is caused by the fact that one circuit has been completed without detecting the target. Therefore, the second circuit should be flown either inside or outside the original peak. This will place the second circuit on one of the modal points of the new distribution. If the second circuit is constructed outside the peak, the third circuit should be constructed inside the peak, so as to be on the other modal value of the distribution.

In order to accomplish this objective the fifth leg was moved a distance S outside the position of the first leg in space relative to the target. This distance S is called the sweep spacing. A complete analytical development of an appropriate value for S is given in Ref. 1, chapter two and chapter seven. The final results of this development obtain the following form for the sweep spacing such that

$$S = 0.75 \sqrt{E\sigma}$$

where

$$E = W/2 \ln 2$$

W = sweep width

= 2 x Sonar Range

σ = the standard deviation of the circular normal distribution.

Therefore, r_5 is determined so that

$$\frac{r_5 + r_4}{S_0} = \frac{r_5 - r_4 - S}{St}$$

so that

$$r_5 = mr_4 + A$$

where

$$A = \frac{S_0 \times S}{S_0 - St}$$

A first approximation to r_1 is St x T. However, the approximation to the spiral by straight search legs requires that r_1 be slightly less that this value. Therefore, a reasonable first approximation to r_1 is obtained by setting $r_1 = 0.9 \times St \times T$. Since any change in target course will reduce the outward component of velocity and since the second circuit is to be outside, the first r_1 was reduced further and taken so that

$$r_1 = 0.8 \times St \times T$$

Continuing we obtain the following:

$$r_2 = mr_1$$

$$r_3 = mr_2$$

$$r_4 = mr_3$$

$$r_5 = mr_4 + A$$

$$r_6 = mr_5$$

$$r_7 = mr_6$$

$$r_8 = mr_7$$

$$r_9 = mr_8 - 2A$$

$$r_{10} = mr_9$$

$$r_{11} = mr_{10}$$

$$r_{12} = mr_{11}$$

$$r_{13} = mr_{12} + 3\ A \text{ etc.}$$

Using the relationship between the lengths of the legs and their distances from the point of fix the following equations were obtained for the various search leg values:

$$L_1 = mr_1$$

$$L_2 = mL_1 + r_1$$

$$L_3 = mL_2$$

$$L_4 = mL_3 + A$$

$$L_5 = mL_4$$

$$L_6 = mL_5 + A$$

$$L_7 = mL_6$$

$$L_8 = mL_7 - 2A$$

$$L_9 = mL_8$$

$$L_{10} = mL_9 - 2A$$

$$L_{11} = mL_{10}$$

$$L_{12} = mL_{11} + 3A \text{ etc.}$$

This search plan was developed for large values of T where it was true that the maximum point of the distribution was located at a distance r = St T. The question arises as to what is the smallest value of T for which this search plan may be used for all values of T larger

than this lower bound without any essential decrease in the probability of detection for a given amount of search effort. By observing figure 1 it is seen that when T is less than σ/St the distribution resembles the original circular normal distribution. For T larger than 2σ/ St the distribution has its maximum at approximately r = St x T and moves outward at St the speed of the target. However, there is a very rapid change in the distribution as T increases from σ/St to 2σ/St. This change of the distribution from a stationary distribution to a distribution moving at the speed of the target makes the decision on a lower bound for T most difficult. Koopman and his associates, by an empirical method of testing the search plan against various values of T, concluded that a reasonable lower bound on T was σ/St. Therefore, for values of T $\geq \sigma$/St this search plan was considered optimal by Koopman and Dobbie. For values of T $< \sigma$/St the plan for large values of T is sufficient and may be used.

C. ANALYSIS OF THE KOOPMAN SEARCH PLAN

It is necessary to evaluate the assumptions used in constructing the Koopman search plan when applied to its use by a submerged submarine searching for another submerged submarine. The assumptions of constant detectability over time and detectability being independent of target aspect are acceptable for the submarine application. However, the searcher obviously is not airborne nor does he possess a considerable speed advantage over the target. The pertinent question is this: Does the inapplicability of this assumption invalidate the search plan as devised by Koopman?

In order to analyze this problem, a computer simulation model was designed. Various combinations of target speed, time late and searcher speed were used to construct the Koopman search plan. By observing figure

two, it is obvious that to be effective against a target whose course is assumed to be uniformly distributed between zero and 360 degrees, a minimum of one circuit or four search legs should be completed. The computer simulation constructed four legs of the search plan for each combination of target speed, searcher speed and time late. The program gave the time required to conduct one circuit of the search plan. In addition, it gives the probability of target detection as a function of actual sonar range as opposed to the estimated sonar range and resulting sweep width used in constructing the search plan. In constructing the simulation model, an option was included to allow the test of another of the basic assumptions of the Koopman search plan, namely, that of constant target speed. The program has two versions of target speed incorporated into it. In the first version the actual target speed is set equal to the estimated target speed used in constructing the search plan. The program can be changed so that the actual target speed is a random variable. In this case, the target speed is drawn from a uniform distribution. The mean of the distribution is the estimated target speed, and the end points of the interval are estimated target speed plus and minus two knots. This allows the evaluation of the search plan, when the assumption of constant target speed is not met, and an analysis of the resulting probabilities of target detection. Table I includes a tabulation of the amount of search time necessary to complete four legs of the Koopman search plan for various combinations of target speed, search speed and times late.

As previously stated, the amount of search time available is always limited. From Table I it is apparent that the amount of time necessary to complete one circuit, herein considered the minimum against a target whose course is uniformly distributed between zero and 360 degrees, is excessively large. The search time is particularly long for the most

probable cases that would be encountered in submarine versus submarine operations.

Tables I and II allow a comparison of the probability of target detection for the case where target speed is deterministic as opposed to the case where it is a random variable.

Because of the excessive amount of time required to complete the minimum number of legs of the Koopman search plan, it was decided to construct an alternate search plan that would not be susceptible to excessively long search times.

III. ALTERNATE SEARCH PLAN

A. BASIC ASSUMPTIONS OF THE ALTERNATE SEARCH PLAN

The Koopman search plan is obviously not universally applicable to submarine versus submarine search due to the excessive search time required to complete even a minimum number of search legs. The cause of this failure is obvious. By examining the multiplicative factor m where

$$m = \frac{S_0 + St}{S_0 - St}$$

the excessive time problem can be identified.

For the range of target speeds and searcher speed combinations used in compiling Tables I and II, the factor m ranges from a high value of 13 to a low value of 1.8. In the original design of the search plan, the searcher was airborne. Searcher speed was generally in the range of 140 to 160 knots. The target, either a surfaced submarine or a surface ship, generally had a top speed of approximately 25 to 30 knots. These two facts taken together show that the multiplier m was severely limited in size. In the submarine versus submarine search problem, this is not the case. By examining the formulae for the various search legs it can be seen that the formulae reduce to a common form such that

$$L_n = m^n r_1 \pm C_n$$

$$\text{where } C_n \geq 0 \, .$$

Therefore, for larger values of the factor m succeeding search legs become exponentially larger. This is the cause of the excessive times necessary to complete a minimum search plan. When target and searcher

speed are relatively close together and moderately large, this effect is even more pronounced. Therefore, an alternative search plan should not be limited in this manner.

Another assumption implicit in the Koopman search plan is that numerous circuits will be completed. This fact was used explicitly in arriving at the form of the r_1 distance. The sweep spacing S was also determined based on the assumption that more than one circuit would be completed. Since the searcher in the submarine application does not enjoy the prerequisite speed advantage to complete many circuits, this assumption should not be used in constructing an alternative search plan.

Intuitively then it seems that in an alternative search plan the multiplicative factor would be relatively "small" so as to allow the completion of the minimum search plan in a reasonable amount of time. Also the r_1 distance would be determined based on the assumption that few circuits could be completed. This is due to the limited amount of search effort available. In many cases tactical considerations will probably limit this time to that necessary to complete one circuit.

B. CONSTRUCTION OF AN ALTERNATE SEARCH PLAN

In order to test an alternative search plan against the Koopman search plan, a computer simulation model was developed. To allow a fair comparison of search plans, the search effort, that is search time, had to be equal for both plans. This time was set arbitrarily at four days (96 hours) for comparison purposes. The target speed was eight knots, the search speed 12 knots, and the time late was selected at four hours. The number of search legs of the Koopman search plan that could be completed in this time was then determined. The number of search legs for the alternate search plan

was likewise determined. The probability of detection as a function of actual sonar range was then calculated for both search plans and compared. In determining the best alternate search plan, it was hypothesized that the construction would be as shown in figure 3.

The distance r_1 would have the general form such that

$$r_1 = (\text{factor}) \times (\text{time late}) \times (\text{estimated target speed})$$

where the factor value was as yet undetermined. The search leg lengths would be obtained as follows:

$$L_n = (\text{multiplier})^n \times r_1$$

where the multiplier value was as yet undetermined. In determining comparison search plans, various combinations of factor values and multiplier values were used. The factor value ranged from 0.8 to 1.5 in 0.1 increments. The multiplier values used were 2.0, 3.0, 4.5 and 6.75. In all these values generated thirty-two different alternate search plans.

The computer program output allowed a direct comparison of each alternative search plan with the Koopman plan. The measure of comparison was the probability of detection as a function of actual sonar range. Although comparison values were available for all sonar ranges from zero to 60 nautical miles, the comparison was made at a sonar range of ten nautical miles. This is the sonar range estimate used in constructing the Koopman search plan values of Tables I and II.

The probability of target detection for different combinations of factor and multiplier values are shown in figure four. In this case, the actual target speed was equal to the estimated target speed. The Koopman search plan yielded a probability of target detection of .0870 under these values of target speed, searcher speed, time late and time available to

search. An alternate search plan using a factor value of 0.9 and a multiplier value of 2.0 produced the highest probability of target detection (.1490). This is considerably better than the results using the Koopman search plan.

However, this is not a fair comparison for the Koopman search plan. Koopman used the approximation for the r_1 offset distance as follows:

$$r_1 = 0.8 \times St \times TL$$

His previous approximation had been

$$r_1 = 0.9 \times St \times TL$$

The distance had been reduced because he felt that the target would not actually be on a constant course. The comparison is not fair because in the simulation model, the target's course is a constant. A better comparison would result if the Koopman search were computed using an r_1 offset of

$$r_1 = 0.9 \times St \times TL$$

based on the fact that the target course is a constant once its initial value is determined. When this is done, the probability of target detection at a sonar range of ten nautical miles is .1140 for the Koopman search plan. Therefore, using a better comparison, the best alternate search plan is still better than the Koopman search plan.

The question arises as to whether or not this particular alternate search plan is best or even better than the Koopman search plan when target speed is a random variable as opposed to being a constant equal to the estimated target speed. The simulation was run again with the actual target speed a random variable. The Koopman search plan yielded the

following values for probability of target detection at a sonar range of ten nautical miles:

$$r_1 = 0.8 \times St \times TL \qquad .0750$$

$$r_1 = 0.9 \times St \times TL \qquad .0990$$

The combination of factor and multiplier values that maximized the probability of target detection at a range of ten nautical miles was the same, that is a factor value of 0.9 and a multiplier value of 2.0. They produced a probability of target detection equal to .1060.

In theory what should now be done is to run the simulation for all interesting values of search speed, target speed and time late. This would be done with a set limit on the amount of search effort available. For each combination of target and search speed, the simulation would have to be run twice--once for the case where the actual target speed equals the estimated target speed and another time for the case where the target speed was a random variable. Each run would produce a different combination of factor value and multiplier value that maximized the probability of target detection. These values might be the same for the case where the target speed is deterministic and the case where it is a random variable or they might be different. One combination of factor and multiplier values might be predominant for different target speed, search speed and time late combinations. This does not necessarily have to be the case.

However, if this is not done, what is the applicability of the alternate search plan that was found best for this particular combination of searcher and target speeds and time lates with other parameter combinations? This hypothesis was tested by using the search leg values that this plan produces in the first computer simulation mentioned. The amount

of time necessary to complete four legs of the search plan for various combinations of search speed, target speed, and time late are tabulated in Table I.

Tables I and II list the probability of target detection for various combinations of search speed, target speed and time late values, using this particular alternate search plan. In the majority of the combinations the alternate search plan produces probabilities of target detection that are greater than the Koopman search plan. This is especially significant in light of the fact that the alternate search plan requires less time to complete one circuit than the Koopman search plan in all but three cases.

Ideally, given a combination of search speed, target speed, and time late, the alternate search plan that maximizes the probability of target detection for this combination should be determined.

It is interesting to note a phenomenon exhibited by both the Koopman search plan and the alternate search plan. For some combinations of target speed, searcher speed and times late, the probability of target detection is higher when the target speed is a random variable than when the target speed is deterministic. This result occurs most often when target speed and search speed are close together and the time late is large.

It is obvious that for some combinations of target initial position and a fixed speed, the target will be undetected by the searcher. However, if the target speed becomes a random variable, the searcher may be able to detect the target. Therefore, in these cases where the search plan has a low probability of target detection, randomness in target speed may increase the probability of target detection.

IV. RECOMMENDATIONS

Since the amount of time required to conduct the minimum number of search legs using the Koopman search plan is excessive, it is recommended that an alternate search plan be used. The particular alternate to be used is dependent on the target speed, searcher speed and time late. The time available to conduct the search is also a deciding factor. The particular search plan to use can be determined by using the computer program described in appendix C. If this is not possible, the alternate search plan using a factor value of 0.9 and a multiplier value of 2.0 could be used. The resulting times to complete one circuit of the search plan and the probabilities of target detection are shown in Tables I and II for various combinations of searcher speed, target speed and time lates.

APPENDIX A

TABLE I

Constants for all Comparisons
One Circuit (Four Legs)
Actual Sonar Range - 10 nmi
Estimated Sonar Range - 10 nmi
Standard Deviation of Circular
Normal Distribution - 2 nmi
Actual Target Speed - Estimated Target Speed

			Koopman Search Plan		Alternate Search Plan	
Time Late (HRS)	Searcher Speed (KTS)	Target Speed (KTS)	Search Time (HRS)	Prob. of Detection	Search Time (HRS)	Prob. of Detection
3	8	4	160.61	.4026	40.50	.4586
3	11	4	40.19	.4153	29.45	.4940
3	14	4	19.20	.4576	23.14	.4150
6	8	4	320.21	.1576	81.00	.1960
6	11	4	79.81	.1880	58.91	.2010
6	14	4	37.99	.2063	46.29	.1383
12	8	4	639.41	.0230	162.00	.0783
12	11	4	159.04	.0213	117.82	.0633
12	14	4	75.59	.0216	92.57	.0600
3	11	8	3416.89	.1240	58.91	.1670
3	14	8	364.47	.1460	46.29	.1983
6	11	8	6832.44	.0216	117.82	.0576
6	14	8	728.27	.0236	92.57	.1006
12	11	8	13,663.53	.0000	235.64	.0096
12	14	8	1,455.87	.0000	185.14	.0273
3	14	12	64,026.36	.0773	69.43	.1190
6	14	12	128,050.69	.0000	138.86	.0656
12	14	12	256,099.63	.0000	277.71	.0123

TABLE II

Constants for all Comparisons
One Circuit (Four Legs)
Actual Sonar Range - 10 nmi
Estimated Sonar Range - 10 nmi
Standard Deviation of Circular Normal Distribution - 2 nmi
Actual Target Speed - Uniform Random Variable

Time Late (HRS)	Searcher Speed (KTS)	Target Speed (KTS)	Koopman Search Plan: Search Time (HRS)	Koopman Search Plan: Prob. of Detection	Alternate Search Plan: Search Time (HRS	Alternate Search Plan: Prob. of Detection
3	8	4	160.61	.3616	40.50	.4033
3	11	4	40.19	.4000	29.45	.4040
3	14	4	19.20	.4313	23.14	.4033
6	8	4	320.21	.1113	81.00	.1266
6	11	4	79.81	.1190	58.91	.1403
6	14	4	37.99	.1486	46.29	.1310
12	8	4	639.41	.0340	162.00	.0320
12	11	4	159.04	.0376	117.82	.0356
12	14	4	75.59	.0453	92.57	.0373
3	11	8	3416.89	.1290	58.91	.1903
3	14	8	364.47	.1373	46.29	.1786
6	11	8	6832.44	.0330	117.82	.0723
6	14	8	728.27	.0456	92.57	.0643
12	11	8	13,663.57	.0086	235.64	.0213
12	14	8	1,455.87	.0166	185.14	.0193
3	14	12	64,026.36	.0810	69.43	.1463
6	14	12	128,050.69	.0190	138.86	.1516
12	14	12	256,099.63	.0026	277.71	.0186

APPENDIX B

This appendix discusses the first of the two computer simulation models used. This simulation determines the amount of time necessary to complete one circuit (four legs) of the Koopman search plan. The output of the simulation is a graph of probability of detection as a function of actual sonar range and a listing of the sixty points plotted by the graph. The program was written in Fortran IV, level G and was run on an IBM 360 digital computer.

The basic program organization is discussed next. The basic input parameters that can be changed are as follows:

ALPHANUMERIC NAME	PARAMETER	VALUES	UNITS
NTRUN	Number of targets run against a particular search plane	>1	
RS	Sonar Range	60>RS>0	NMI
SIGMA	Standard deviation of circular normal distribution	>0	NMI
SO	Initial value of searcher speed	ST < SO≤14	KTS
STE	Initial value of target speed estimate	0<ST<SO	KTS
TL	Initial value of time late	0<TL≤12.0	HRS
ULCT	Upper limit on target course	0<ULCT≤2Π	rad
LLCT	Lower limit on target course	0≤LLCT<2Π	rad

For this program the input parameters mentioned above were initialized as follows:

NTRUN = 3000

RS = 10.0

SIGMA = 2.0

SO = 8.0

STE = 4.0

TL = 4.0

ULCT = 2π

LLCT = 0.0

The program then uses these initial values to compute the parameters necessary for constructing the Koopman expending square search plan. The four search leg lengths are then computed. These distances and the r_1 distance are then used to establish the (x, y) co-ordinates for each search leg end point. Using the search speed input, the time at which each of these co-ordinates is reached is determined as is the total search time necessary for the entire search plan. The headings entitled initial values for search legs one, two, three and four will be discussed after the determination of target motion is discussed. The target course is then obtained from a uniform distribution. The end points of this interval are variable as previously mentioned. In this case the target speed was selected uniformly between values of zero and 360 degrees. The target speed is then obtained from a uniform distribution. The actual target speed can be set at the estimated target speed by using the following input cards:

ULST = STE

LLST = STE

Or the actual target speed can be a random variable drawn from a uniform distribution whose end points are the estimated target speed plus or minus two knots. This is done by substituting the following cards in the input deck:

ULST = STE + 2.0

LLST = STE - 2.0

The direction and distance of the uncertainty in target position from the origin (point of fix) is then generated from a circular normal distribution. The program then uses the randomly selected target course to determine the target's velocity components in the x and y directions. Using the random target position uncertainty selected from the circular normal distribution, the initial (x, y) coordinates of the target are then determined.

The conceptual framework associated with the next section of the program is discussed in detail in this section. For each randomly selected target track there will be some time at which the target and searcher are at a minimum range. This range is called the closest point of approach or CPA range.

It is useful to think of each search leg as being extended in space past the point where the turn to the next search leg is made. At any time on any search leg the searcher's position is a function of time and his speed. That is to say:

$$(x_s, y_s) = (a_1 + a_2 t, a_3 + a_4 t)$$

where

a_1 = search leg initial x- coordinate

a_3 = search leg initial y- coordinate

a_2 = searcher x- velocity component

a_4 = searcher y- velocity component

In a similar manner the target's position can be expressed as follows:

$$(x_t, y_t) = (b_1 + b_2 t \ , \ b_3 + b_4 t)$$

where

b_1 = target's initial x- coordinate

b_3 = target's initial y- coordinate

b_2 = target's x- velocity component

b_4 = target's y- velocity component

It also can be shown that the searcher/target range is obtained as follows:

$$\text{Range} = \left[(x_s - x_t)^2 + (y_s - y_t)^2\right]^{1/2}$$

Now to minimize this range the derivative of the equation with respect to time is taken and then is set equal to zero. This procedure yields the time at which the CPA range occurs. This time is given by the following equation:

$$t_{rcpa} = \frac{(a_1 - b_1)\ (a_2 - b_2) + (a_3 - b_3)\ (a_4 - b_4)}{(a_2 - b_2)^2 + (a_4 - b_4)^2}$$

If this time actually occurs while the searcher is traversing this search leg, the range at this time is a candidate for a minimum range. If the minimum range on this search leg does not occur on the interior of the search leg interval, then it must be an end point solution. The target/searcher range, when the searcher is at the end points of each search leg, is then easily determined.

The target's speed is a constant once selected. Therefore, the x,y velocity components are constants. The target's initial position is also constant once determined. On a particular search leg the searcher x,y velocity components are constants. One is identically zero, depending

on the orientation of the search leg. Therefore, for each search leg there are at most three candidates for the CPA range and at the least two. If the solution to the time equation falls within the time frame that the searcher is on this particular search leg, then there are three candidates for a minimum on this search leg. These three are the search leg end point ranges and the range at the time equation solution. If the time value that satisfies the equation occurs when the searcher is not on this particular search leg then there are only two candidates for a minimum range, the search leg end point ranges. Using this procedure all possible candidates for minima can be obtained for the entire search plan. Once the CPA range for the iteration has been determined, it is placed in a tally box. There are sixty tally boxes. Each tally box corresponds to an interval of one nautical mile. Once the program has determined the CPA range for each target, this range is placed in the tally box for the interval into which it falls. All CPA ranges in excess of sixty nautical miles were placed in the last tally box. The program tests at this point to determine if the required number of targets have been run against this particular search plan. If not, the program generates another set of target parameters and repeats the above mentioned step. If the required number of runs has been obtained, the program sums all the tally box values and then determines the probability that the CPA range will fall in the interval of each of the individual tally boxes. This information allows the calculation of the probability that the CPA range will be less than or equal to a particular range. In essence this is the probability of target detection as a function of the actual sonar range. The output of the program is twofold. First, a print out of the probability of target detection as a function of sonar range is provided.

These values are listed for every nautical mile between one and sixty nautical miles. Secondly, the probability of target detection as a function of sonar range is plotted graphically for values of sonar range from zero to sixty nautical miles. The program then varies the searcher speed, target speed and time late to obtain all the combinations shown in Tables I and II.

The probability of target detection as a function of actual sonar range is in fact an empirical cumulative distribution function. The number of iterations performed of different target tracks (3000) ensures that the empirical cumulative distribution function will be within $\pm$ .025 units of the population cumulative distribution function. This is with a probability of .95, that is to say $\alpha = .05$. The exact number of runs needed was arrived at using Kolmogorov's statistic. These values are tabulated in Table A21-b of Ref. 2.

The pseudorandom number generator used was tested for randomness using a separate computer program not described in the thesis. The statistical tests for randomness used were the frequency test, serial test, lagged product test, runs up and down test, and runs above and below the mean test. The pseudorandom number test passes all of these tests adequately, thereby justifying its use.

It should be noted that by replacing the r_1 distance calculation and the search leg length computations for the Koopman plan by those necessary for the alternate search plan, the same output can be obtained for the alternate search plan. The following card must be removed:

$$SM = (SO + STE)/(SO - STE)$$

$$r_1 = 0.8 * STE * TL$$

$$SL1 = SM * r_1$$

$$SL2 = SM * SL1 + r_1$$

$$SL3 = SM * SL2$$

$$SL4 = SM * SL3 + A$$

Once these cards have been removed the following cards have to be inserted in their places:

$$SM = 2.0$$

$$r_1 = 0.9 * STE * TL$$

$$SL1 = SM * r_1$$

$$SL2 = SM ** 2 * r_1$$

$$SL3 = SM ** 3 * r_1$$

$$SL4 = SM ** 4 * r_1$$

The program will then determine the values depicted by Tables I and II for the alternate search plan that was determined to be best.

APPENDIX C

This appendix describes the second computer simulation used in the analysis and comparison of search plans. The primary purpose of this simulation is to compare alternate search plans with the Koopman search plan. Therefore, the goal of this simulation is to compare search plans which require the same amount of search effort, that is search time.

With this goal in mind, the input variables were chosen as follows:

ALPHANUMERIC NAME	PARAMETER	VALUES	UNITS
STE	Estimated target speed	0<STE<SO	KTS
SO	Searcher speed	STE<SO	KTS
TL	Time late	>0	HRS
TLimit	Amount of search time available	>0	HRS
NTRUNS	Number of target runs against a particular search plan	>0	
ULCT	Upper limit target course	$0<ULCT\leq 2\pi$	RADIANS
LLCT	Lower limit target course	$0\leq LLCT<2\pi$	RADIANS
SIGMA	Standard deviation of circular normal distribution	>0	NMI
RS	Sonar Range	>0	NMI

The program was run with the following initial values for these parameters:

STE = 8.0

SO = 12.0

TL = 4

NTRUNS = 3000

ULCT = 2π

LLCT = 0.0

SIGMA = 2.0

RS = 10.0

The program input considered next is the actual target speed input. There are two options available. First the following two cards may be used:

ULST = STE

LLST = STE

When used, these cards result in the actual target speed being a constant equal to the estimated target speed. The second option involves inserting the following cards into the deck:

ULST = STE + 2.0

LLST = STE - 2.0

This results in the actual target speed for each iteration being drawn from a uniform distribution. The mean of the distribution is the estimated target speed while the end points are the estimated target speed plus and minus two knots. The program then computes the parameters necessary for the calculation of the Koopman search plan. The (x,y) coordinate values for each of these search leg end points is then calculated. This having been done, the values for twenty search legs were computed. The values necessary to determine the searcher's initial position and velocity component for each of the search legs is computed. The program then determines the time that the searcher will reach each of the search leg end points. The procedure stops the first instance in which this time exceeds the limit of the search time available. By so doing, the number of search legs or parts of search legs that can be completed in the search time available has been determined. The target's course is then selected from a uniform distribution. The limits of this distribution are variable and have been

previously mentioned. The target's position uncertainty is then selected from a circular normal distribution. The next target parameter determined is target speed. This speed comes from a uniform distribution as mentioned before.

The target's initial (x,y) coordinates are then determined. The target's velocity components in the x,y direction are calculated next. The candidates for the minimum target/searcher range are then calculated using the procedure fully explained in appendix A. From these a CPA range for this target iteration is determined. This range is placed in a tally box and the procedure is repeated until the required number of target iterations has been performed. When this has occurred, the probability of target detection as a function of sonar range is calculated.

The input parameters for the alternate search plan are now listed. They are as follows:

$$\text{OMULT} = 2.0$$

$$\text{FACTOR} = 0.8$$

The factor value is used in determining the r_1 distance as follows:

$$r_1 = \text{FACTOR} * \text{STE} * \text{TL}$$

and the quantity OMULT is used in determining the search leg lengths where

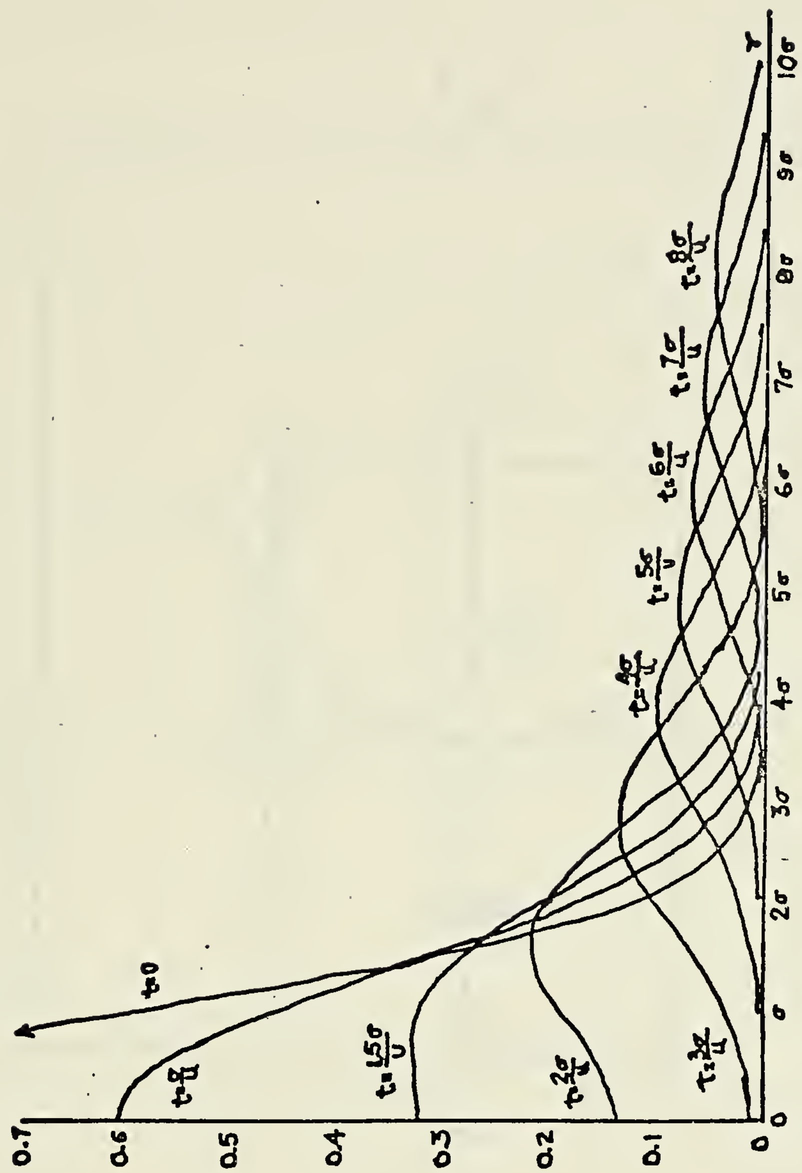
$$L_n = \text{OMULT}^n (r_1)$$

The value of twenty-four search leg's lengths for each FACTOR and OMULT combination are then determined. The (x,y) coordinates values for each search leg are calculated. The parameters necessary for the target/searcher minimum range calculation are determined at this point. The number of full and partial search legs that the target can complete in the allotted search time is then determined. The target course, speed

and position uncertainty values are then determined. The closest point of approach is determined for this iteration. This CPA range is placed in a tally box. The program tests to see if the desired number of targets have been run for this search plan. If they have not, it generates targets until the required number has been reached. For each target it computes the CPA range and places it in a tally box. The program then computes the values of probability of target detection as a function of sonar range.

These values are then printed for the Koopman search plan and the particular alternate search plan under consideration. The program then plots the graph of probability of detection as a function of sonar range. The values for the Koopman search plan and the particular alternate search plan under consideration are then plotted on the same graph. This is done to facilitate comparison. The factor value used in the r_1 determination and then the multiplier value used in the search leg length calculation are then varied. The entire alternate search is then recalculated. The required number of targets are run against this search plan and the probability of detection as a function of sonar range is calculated. Once these values are calculated they are printed out and plotted for easy comparison with the Koopman search plan.

APPENDIX D



Distance r from point of fix

Distribution of a moving target about a point of fix o

figure 1

KOOPMAN SEARCH PLAN

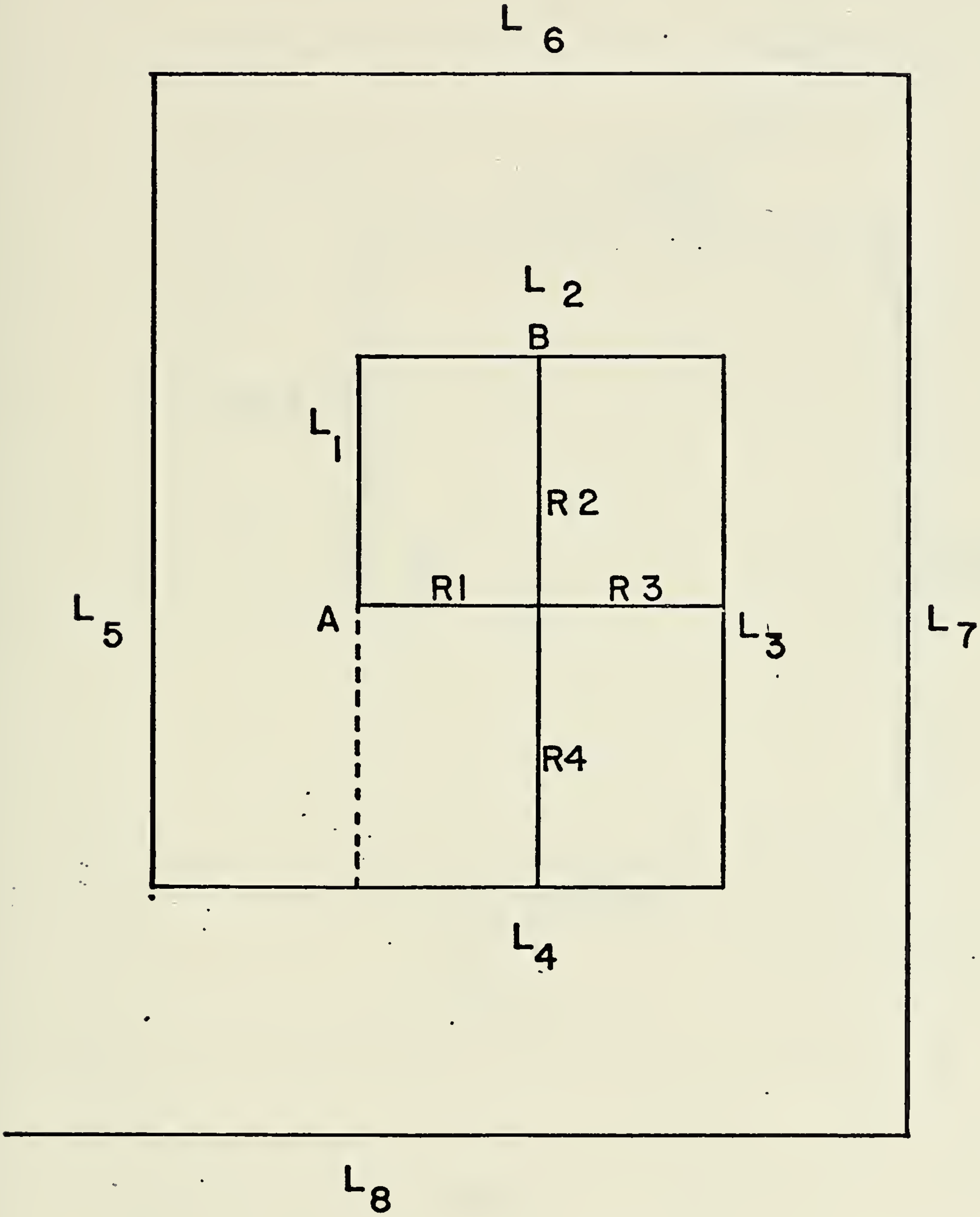


figure 2

ALTERNATE SEARCH PLAN

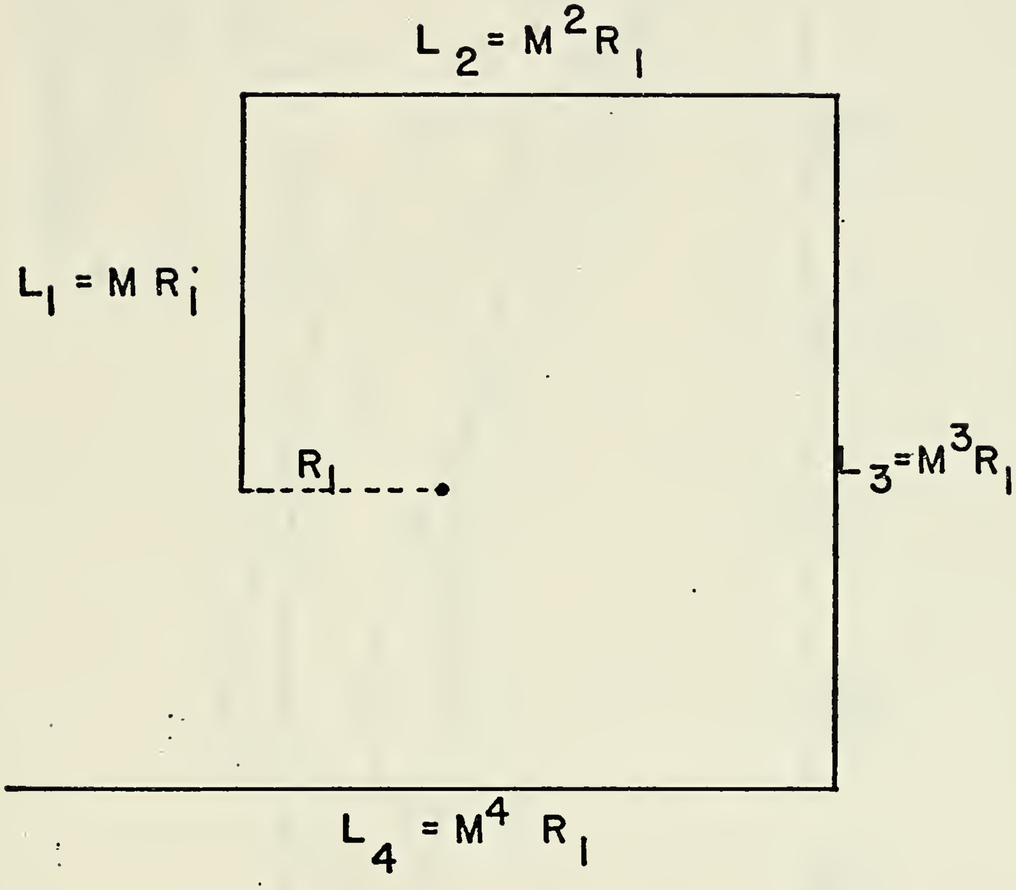


figure 3

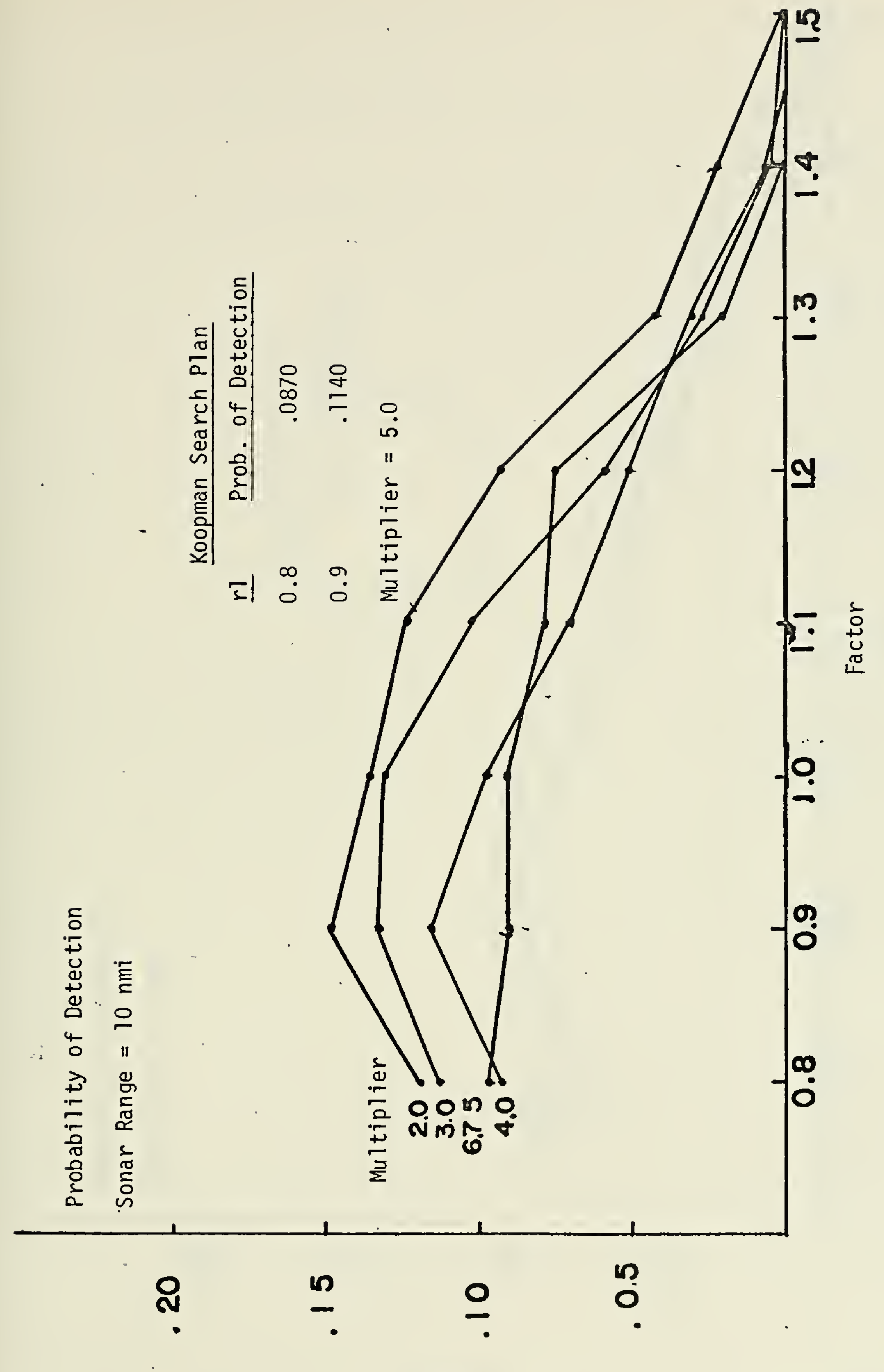
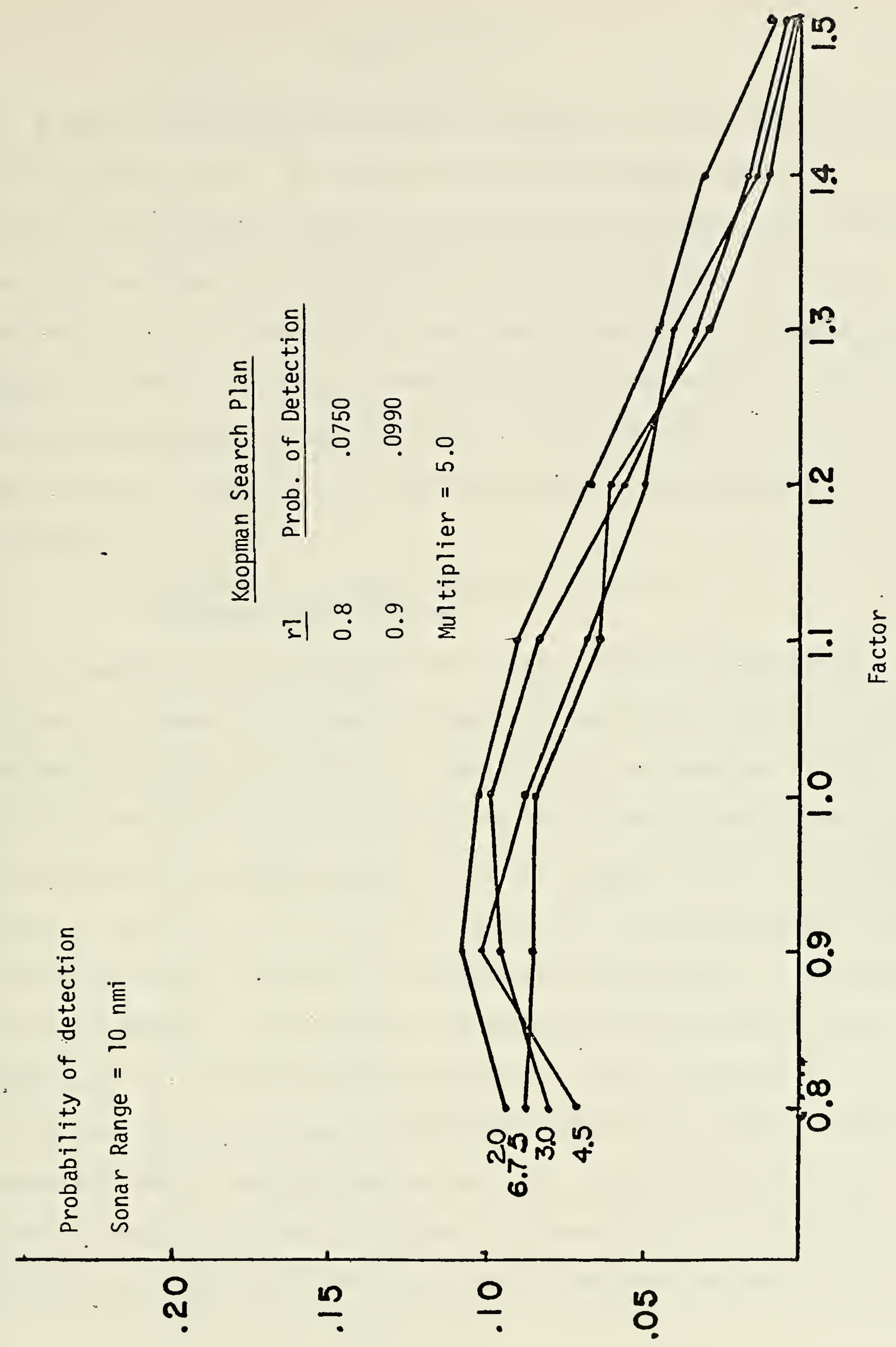

Probability of Detection
Sonar Range = 10 nmi
Koopman Search Plan
rl
Prob. of Detection
0.8
.0870
0.9
.1140
Multiplier = 5.0
Multiplier
2.0
3.0
6.75
4.0
.20
.15
.10
.05
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
Factor

figure 4



Probability of detection
Sonar Range = 10 nmi
Koopman Search Plan
r1 Prob. of Detection
0.8 .0750
0.9 .0990
Multiplier = 5.0
.20
.15
.10
.05
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
Factor
2.0
6.75
3.0
4.5

figure 5

COMPUTER OUTPUT

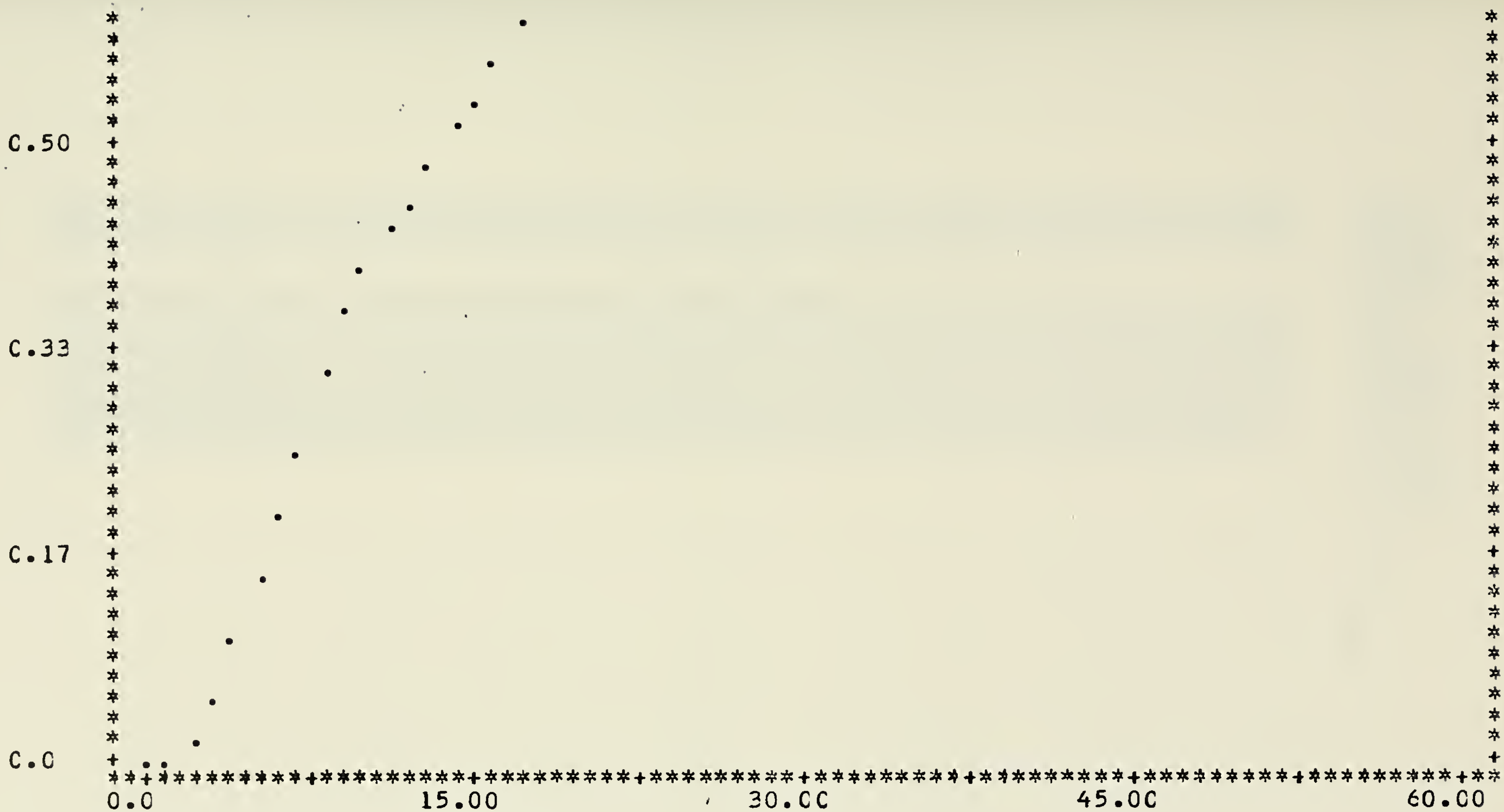
A sample of the computer output is included in this section for each of the programs used. The output for the first program consists of a listing of the probabilities of target detection for actual sonar ranges for a limited number of target speeds, searcher speeds and times late combinations. After each listing, the same information is presented in a graphical format. The x-axis represents actual sonar range in nautical miles, and the y-axis is the probability of target detection. In using the listing, a computer design feature made the following convention necessary:

Probability of Target Detection = CDF(x+1)
for Sonar Range of x nmi

The output of the second computer program consists of comparative listings and comparative graphs for the input parameters specified in the program explanation of Appendix C. The listings and graphical results are given for the Koopman search plan and the alternate search plans. The alternate search plans used a multiplier value of 2.0 and factor values of 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, and 1.5. The same convention is used for the listings. The axes of the graphs are also the same. In the graphical representation of the results, the Koopman search plan values are denoted by "." and the alternate search plan values are denoted by "+". In the case where the values of the Koopman search plan and the alternate search plan under consideration are the same, only the value for the alternate search plan (+) is printed. The exact probability of target detection at a particular sonar range can be obtained from the listings.

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED= 8.0
ASSUMED TARGET SPEED= 4.0
TIME LATE= 3.0
TOTAL SEARCH TIME =     163.61


CDF    1=0.0
CDF    2=0.00300
CDF    3=0.01567
CDF    4=0.04967
CDF    5=0.10033
CDF    6=0.15333
CDF    7=0.20800
CDF    8=0.25667
CDF    9=0.30933
CDF   10=0.36267
CDF   11=0.40267
CDF   12=0.42933
CDF   13=0.45333
CDF   14=0.48200
CDF   15=0.51033
CDF   16=0.53967
CDF   17=0.57267
CDF   18=0.60700
CDF   19=0.66567
CDF   20=0.72700
CDF   21=0.80833
CDF   22=0.88433
CDF   23=0.95633
CDF   24=0.98567
CDF   25=0.99433
CDF   26=0.99933
CDF   27=1.00000
CDF   28=1.00000
CDF   29=1.00000
CDF   30=1.00000
CDF   31=1.00000
CDF   32=1.00000
CDF   33=1.00000
CDF   34=1.00000
CDF   35=1.00000
CDF   36=1.00000
CDF   37=1.00000
CDF   38=1.00000
CDF   39=1.00000
CDF   40=1.00000
CDF   41=1.00000
CDF   42=1.00000
CDF   43=1.00000
CDF   44=1.00000
CDF   45=1.00000
CDF   46=1.00000
CDF   47=1.00000
CDF   48=1.00000
CDF   49=1.00000
CDF   50=1.00000
CDF   51=1.00000
CDF   52=1.00000
CDF   53=1.00000
CDF   54=1.00000
CDF   55=1.00000
CDF   56=1.00000
CDF   57=1.00000
CDF   58=1.00000
CDF   59=1.00000
CDF   60=1.00000
CDF   61=1.00000
```



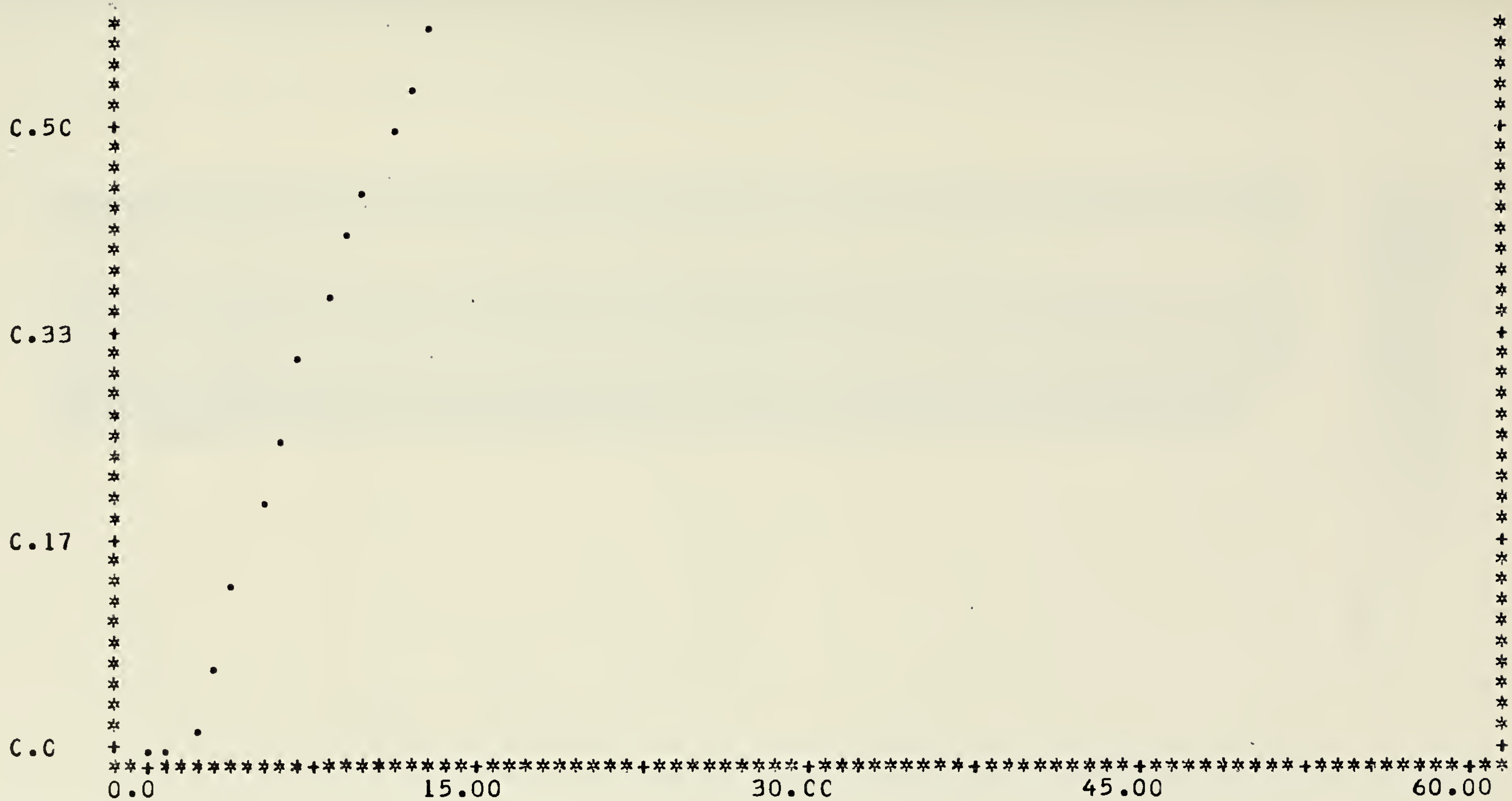
X-SCALE: "*"= 0.75CE CO UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARAMETERS
SEARCHER SPEED= 8.0
, ASSUMED TARGET SPEED= 4.0
TIME LATE= 3.0

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.0
TIME LATE= 3.0
TOTAL SEARCH TIME =      22.20


CDF    1=0.0
CDF    2=0.00533
CDF    3=0.02200
CDF    4=0.06600
CDF    5=0.12833
CDF    6=0.19333
CDF    7=0.25467
CDF    8=0.32100
CDF    9=0.37333
CDF   10=0.41833
CDF   11=0.45767
CDF   12=0.49667
CDF   13=0.53833
CDF   14=0.58333
CDF   15=0.62500
CDF   16=0.68400
CDF   17=0.74400
CDF   18=0.79967
CDF   19=0.85167
CDF   20=0.88167
CDF   21=0.91067
CDF   22=0.94467
CDF   23=0.97967
CDF   24=0.99133
CDF   25=0.99733
CDF   26=0.99867
CDF   27=1.00000
CDF   28=1.00000
CDF   29=1.00000
CDF   30=1.00000
CDF   31=1.00000
CDF   32=1.00000
CDF   33=1.00000
CDF   34=1.00000
CDF   35=1.00000
CDF   36=1.00000
CDF   37=1.00000
CDF   38=1.00000
CDF   39=1.00000
CDF   40=1.00000
CDF   41=1.00000
CDF   42=1.00000
CDF   43=1.00000
CDF   44=1.00000
CDF   45=1.00000
CDF   46=1.00000
CDF   47=1.00000
CDF   48=1.00000
CDF   49=1.00000
CDF   50=1.00000
CDF   51=1.00000
CDF   52=1.00000
CDF   53=1.00000
CDF   54=1.00000
CDF   55=1.00000
CDF   56=1.00000
CDF   57=1.00000
CDF   58=1.00000
CDF   59=1.00000
CDF   60=1.00000
CDF   61=1.00000
```



X-SCALE: "*"= 0.75CE 00 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARAMETERS
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.C
TIME LATE= 3.0

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED= 8.0
ASSUMED TARGET SPEED= 4.0
TIME LATE= 6.0
TOTAL SEARCH TIME =    326.21


CDF    1=0.0
CDF    2=0.0
CDF    3=0.0
CDF    4=0.00233
CDF    5=0.00900
CDF    6=0.02700
CDF    7=0.04933
CDF    8=0.07533
CDF    9=0.10067
CDF   10=0.13300
CDF   11=0.15767
CDF   12=0.18067
CDF   13=0.20567
CDF   14=0.23100
CDF   15=0.25700
CDF   16=0.28000
CDF   17=0.30333
CDF   18=0.32733
CDF   19=0.34800
CDF   20=0.36800
CDF   21=0.38500
CDF   22=0.39833
CDF   23=0.41233
CDF   24=0.42267
CDF   25=0.43733
CDF   26=0.45167
CDF   27=0.46667
CDF   28=0.47867
CDF   29=0.49100
CDF   30=0.50667
CDF   31=0.51700
CDF   32=0.53400
CDF   33=0.55233
CDF   34=0.56967
CDF   35=0.59067
CDF   36=0.62033
CDF   37=0.65400
CDF   38=0.68467
CDF   39=0.71500
CDF   40=0.75533
CDF   41=0.79800
CDF   42=0.84067
CDF   43=0.89067
CDF   44=0.94533
CDF   45=0.97633
CDF   46=0.99000
CDF   47=0.99600
CDF   48=0.99933
CDF   49=0.99967
CDF   50=0.99967
CDF   51=1.00000
CDF   52=1.00000
CDF   53=1.00000
CDF   54=1.00000
CDF   55=1.00000
CDF   56=1.00000
CDF   57=1.00000
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CDF   60=1.00000
CDF   61=1.00000
```

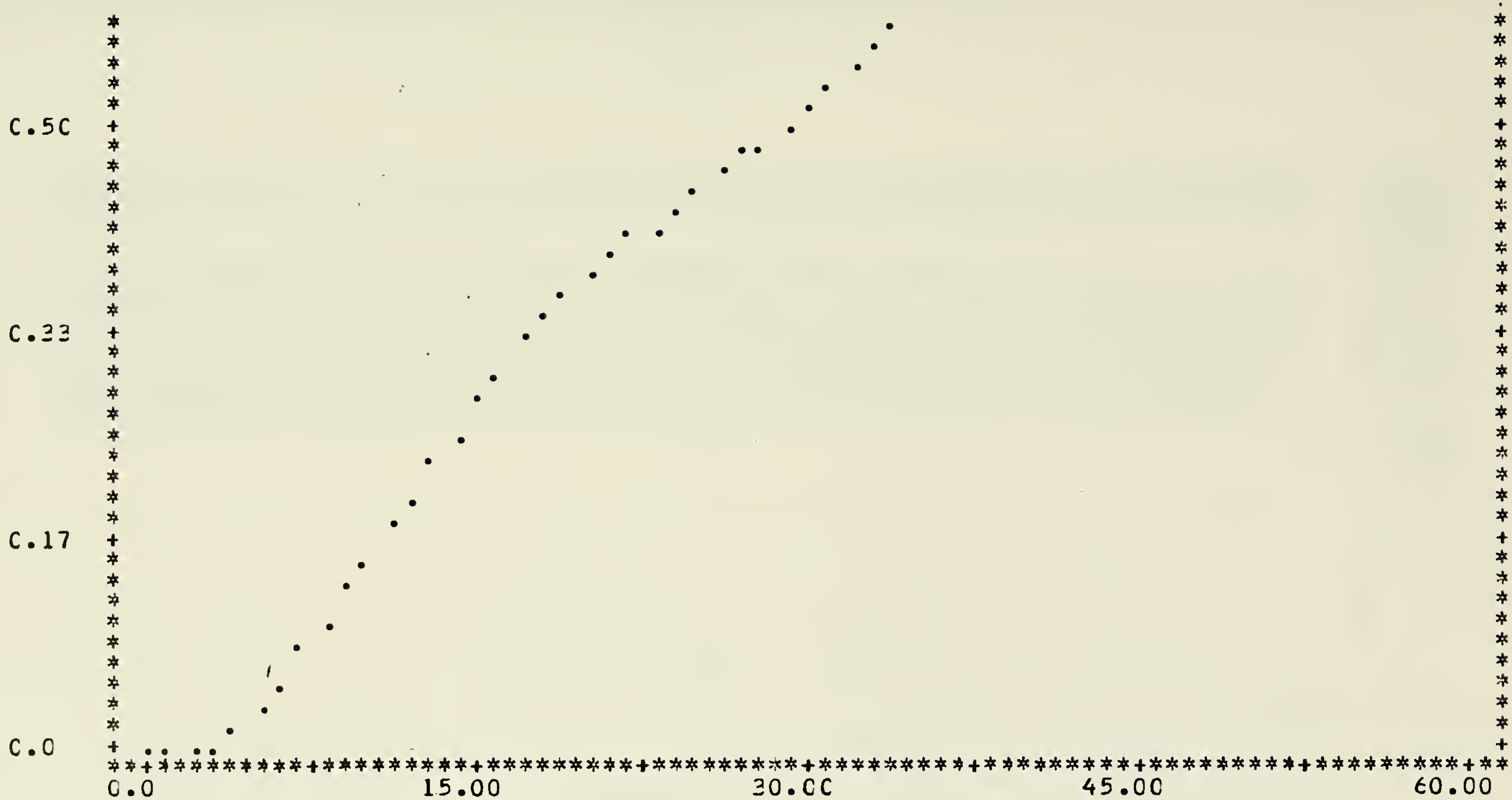

X-SCALE: "*"= 0.75CE CO UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARAMETERS
SEARCHER SPEEC= 8.0
ASSUMEC TARGET SPEED= 4.0
TIME LATE= 6.C

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.0
TIME LATE= 6.0
TOTAL SEARCH TIME =      43.99

CDF    1=0.0
CDF    2=0.0
CDF    3=0.00100
CDF    4=0.00433
CDF    5=0.01233
CDF    6=0.03100
CDF    7=0.06833
CDF    8=0.10900
CDF    9=0.14367
CDF   10=0.17867
CDF   11=0.20633
CDF   12=0.22967
CDF   13=0.25867
CDF   14=0.29467
CDF   15=0.32633
CDF   16=0.36100
CDF   17=0.38667
CDF   18=0.41100
CDF   19=0.43367
CDF   20=0.45500
CDF   21=0.47333
CDF   22=0.49767
CDF   23=0.52000
CDF   24=0.53933
CDF   25=0.55800
CDF   26=0.58033
CDF   27=0.59833
CDF   28=0.62133
CDF   29=0.64867
CDF   30=0.66833
CDF   31=0.69367
CDF   32=0.72467
CDF   33=0.75367
CDF   34=0.78467
CDF   35=0.81500
CDF   36=0.84033
CDF   37=0.86067
CDF   38=0.87167
CDF   39=0.87867
CDF   40=0.88700
CDF   41=0.90267
CDF   42=0.92633
CDF   43=0.95100
CDF   44=0.97800
CDF   45=0.99200
CDF   46=0.99633
CDF   47=0.99900
CDF   48=0.99933
CDF   49=0.99967
CDF   50=1.00000
CDF   51=1.00000
CDF   52=1.00000
CDF   53=1.00000
CDF   54=1.00000
CDF   55=1.00000
CDF   56=1.00000
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CDF   58=1.00000
CDF   59=1.00000
CDF   60=1.00000
CDF   61=1.00000
```
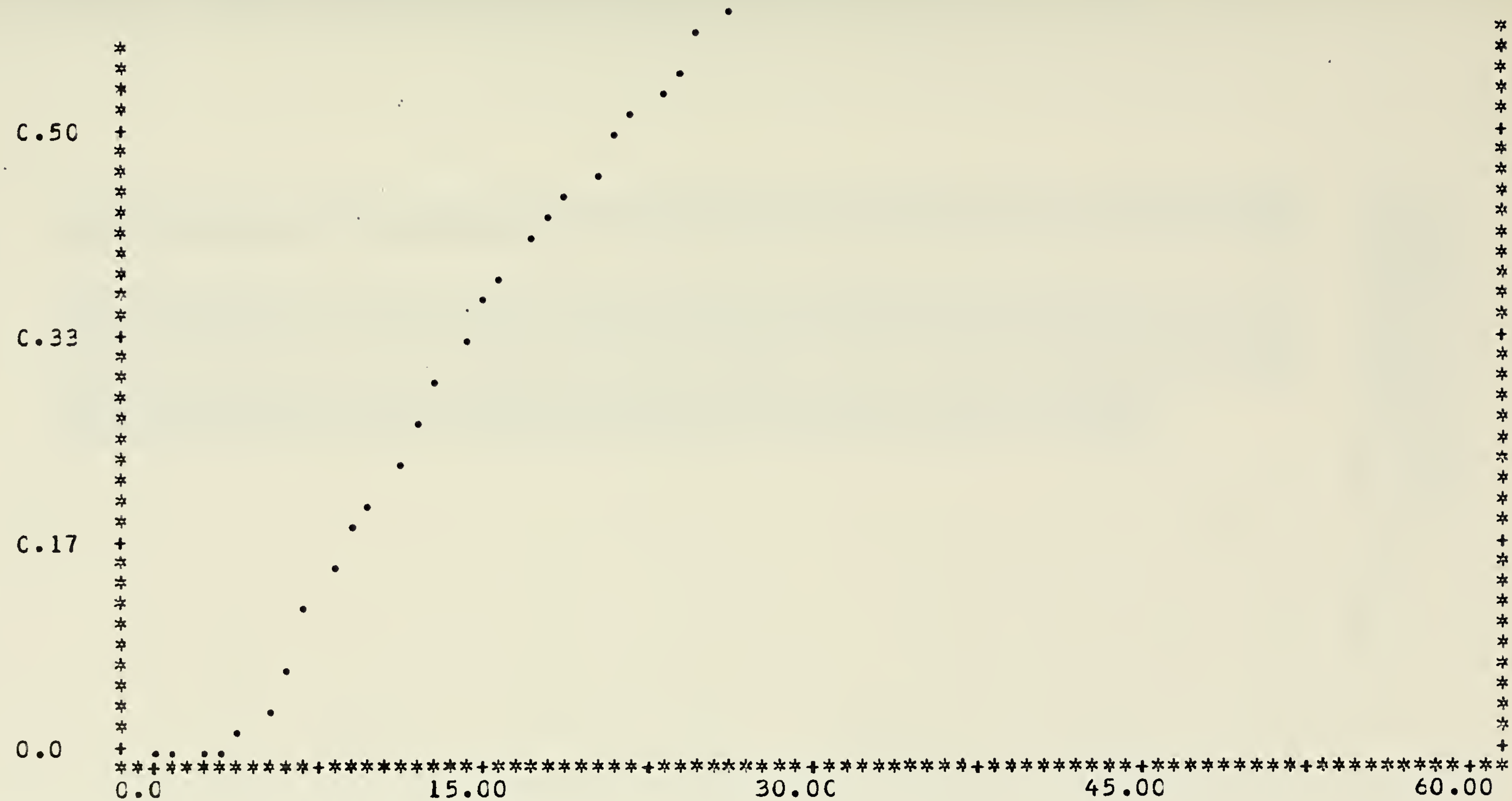
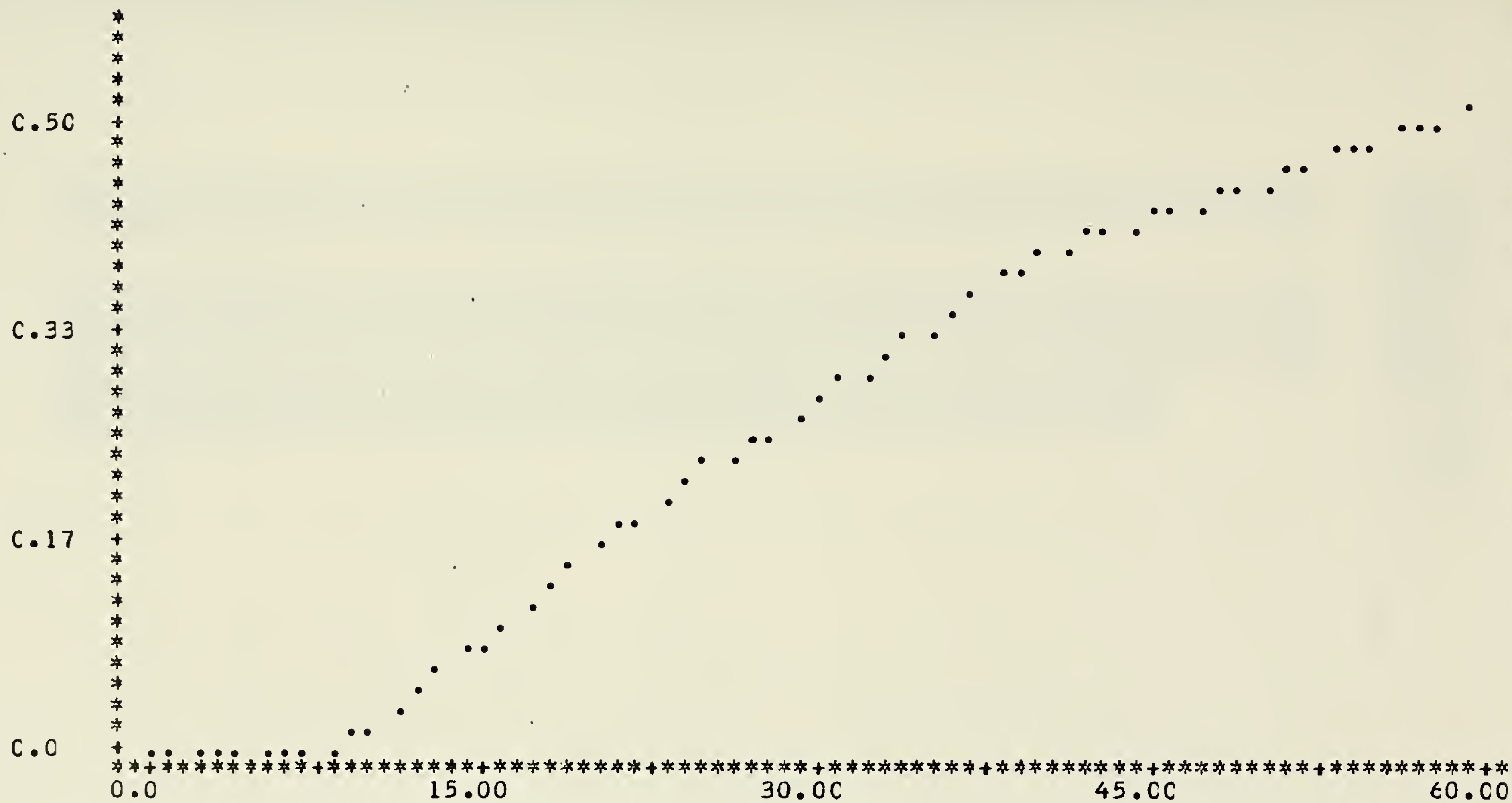



X-SCALE: "*"= C.75CE 00 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARAMETERS
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.0
TIME LATE= 6.C

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED= 8.0
ASSUMED TARGET SPEED= 4.0
TIME LATE=12.0
TOTAL SEARCH TIME =     651.41


CDF   1=0.0
CDF   2=0.0
CDF   3=0.0
CDF   4=0.0
CDF   5=0.0
CDF   6=0.0
CDF   7=0.0
CDF   8=0.00133
CDF   9=0.00333
CDF  10=0.00867
CDF  11=0.02300
CDF  12=0.03533
CDF  13=0.04600
CDF  14=0.06100
CDF  15=0.07700
CDF  16=0.08833
CDF  17=0.10267
CDF  18=0.12133
CDF  19=0.13633
CDF  20=0.15233
CDF  21=0.16700
CDF  22=0.17567
CDF  23=0.18567
CDF  24=0.19900
CDF  25=0.21100
CDF  26=0.22533
CDF  27=0.23567
CDF  28=0.24567
CDF  29=0.25633
CDF  30=0.26700
CDF  31=0.28033
CDF  32=0.29400
CDF  33=0.30467
CDF  34=0.31733
CDF  35=0.33033
CDF  36=0.34000
CDF  37=0.35033
CDF  38=0.36267
CDF  39=0.37767
CDF  40=0.38833
CDF  41=0.39767
CDF  42=0.40433
CDF  43=0.41100
CDF  44=0.41633
CDF  45=0.42233
CDF  46=0.42833
CDF  47=0.43467
CDF  48=0.43867
CDF  49=0.44400
CDF  50=0.45000
CDF  51=0.45733
CDF  52=0.46433
CDF  53=0.47000
CDF  54=0.47667
CDF  55=0.48533
CDF  56=0.48900
CDF  57=0.49400
CDF  58=0.50000
CDF  59=0.50733
CDF  60=0.51367
CDF  61=1.00000
```

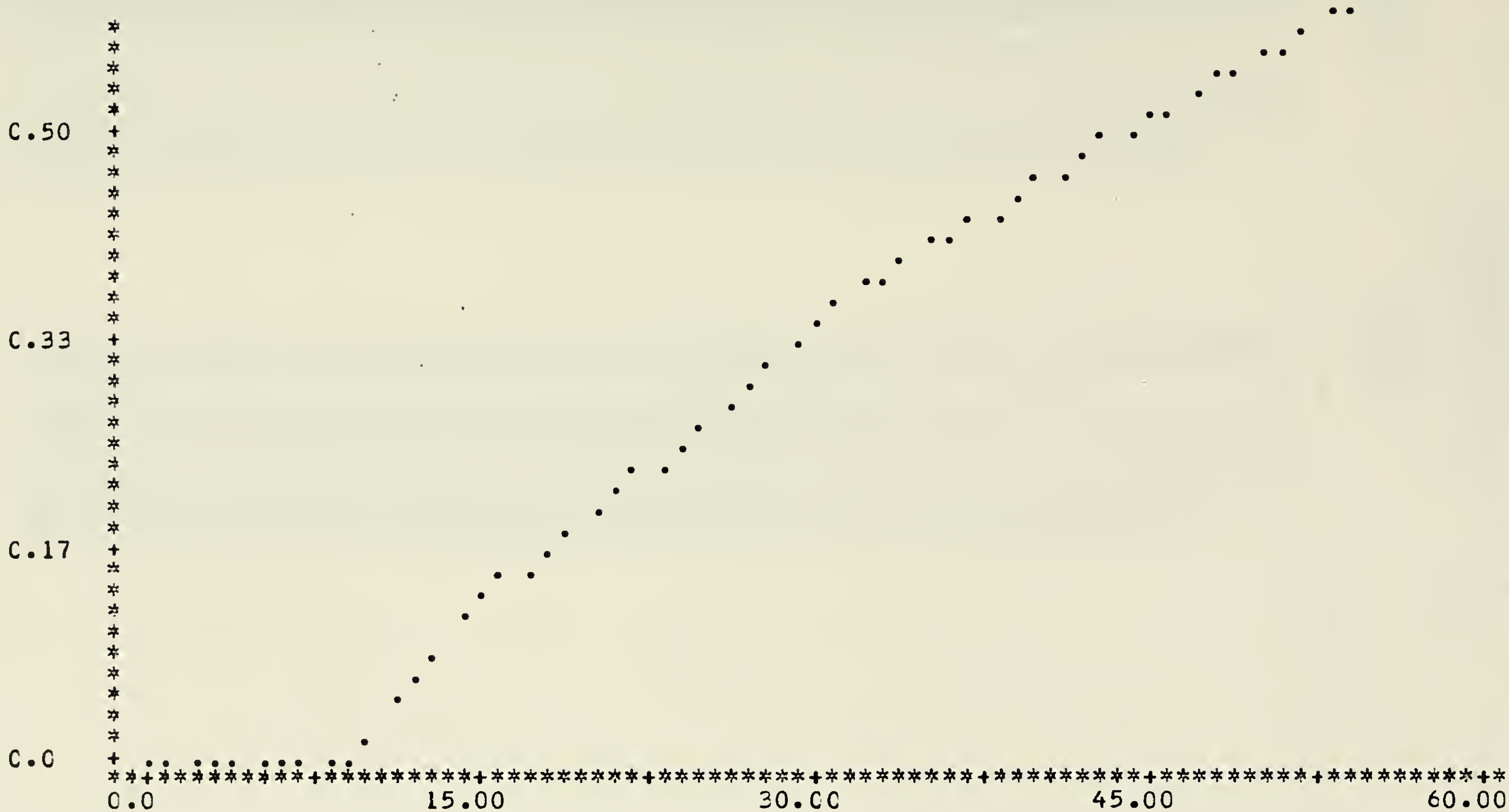


X-SCALE: "*"= 0.75CE C0 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLCWING PARAMETERS
SEARCHER SPEED= 8.0
ASSUMED TARGET SPEED= 4.0
TIME LATE=12.C

```
KOOPMAN SEARCH PLAN
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.0
TIME LATE=12.0
TOTAL SEARCH TIME =      87.58


CDF    1=0.0
CDF    2=0.0
CDF    3=0.0
CDF    4=0.0
CDF    5=0.0
CDF    6=0.0
CDF    7=0.0
CDF    8=0.0
CDF    9=0.00333
CDF   10=0.00767
CDF   11=0.02167
CDF   12=0.04200
CDF   13=0.06533
CDF   14=0.08833
CDF   15=0.10900
CDF   16=0.12700
CDF   17=0.14300
CDF   18=0.15767
CDF   19=0.17367
CDF   20=0.18633
CDF   21=0.20000
CDF   22=0.21400
CDF   23=0.22600
CDF   24=0.24000
CDF   25=0.25400
CDF   26=0.26833
CDF   27=0.28200
CDF   28=0.29400
CDF   29=0.31267
CDF   30=0.33367
CDF   31=0.35067
CDF   32=0.36467
CDF   33=0.37767
CDF   34=0.38933
CDF   35=0.40300
CDF   36=0.41533
CDF   37=0.42400
CDF   38=0.43267
CDF   39=0.44133
CDF   40=0.44867
CDF   41=0.46033
CDF   42=0.47033
CDF   43=0.48133
CDF   44=0.49267
CDF   45=0.50267
CDF   46=0.51033
CDF   47=0.52033
CDF   48=0.53300
CDF   49=0.54267
CDF   50=0.55033
CDF   51=0.56033
CDF   52=0.57133
CDF   53=0.58333
CDF   54=0.59500
CDF   55=0.60600
CDF   56=0.62133
CDF   57=0.63567
CDF   58=0.64500
CDF   59=0.66100
CDF   60=0.67867
CDF   61=1.00000
```



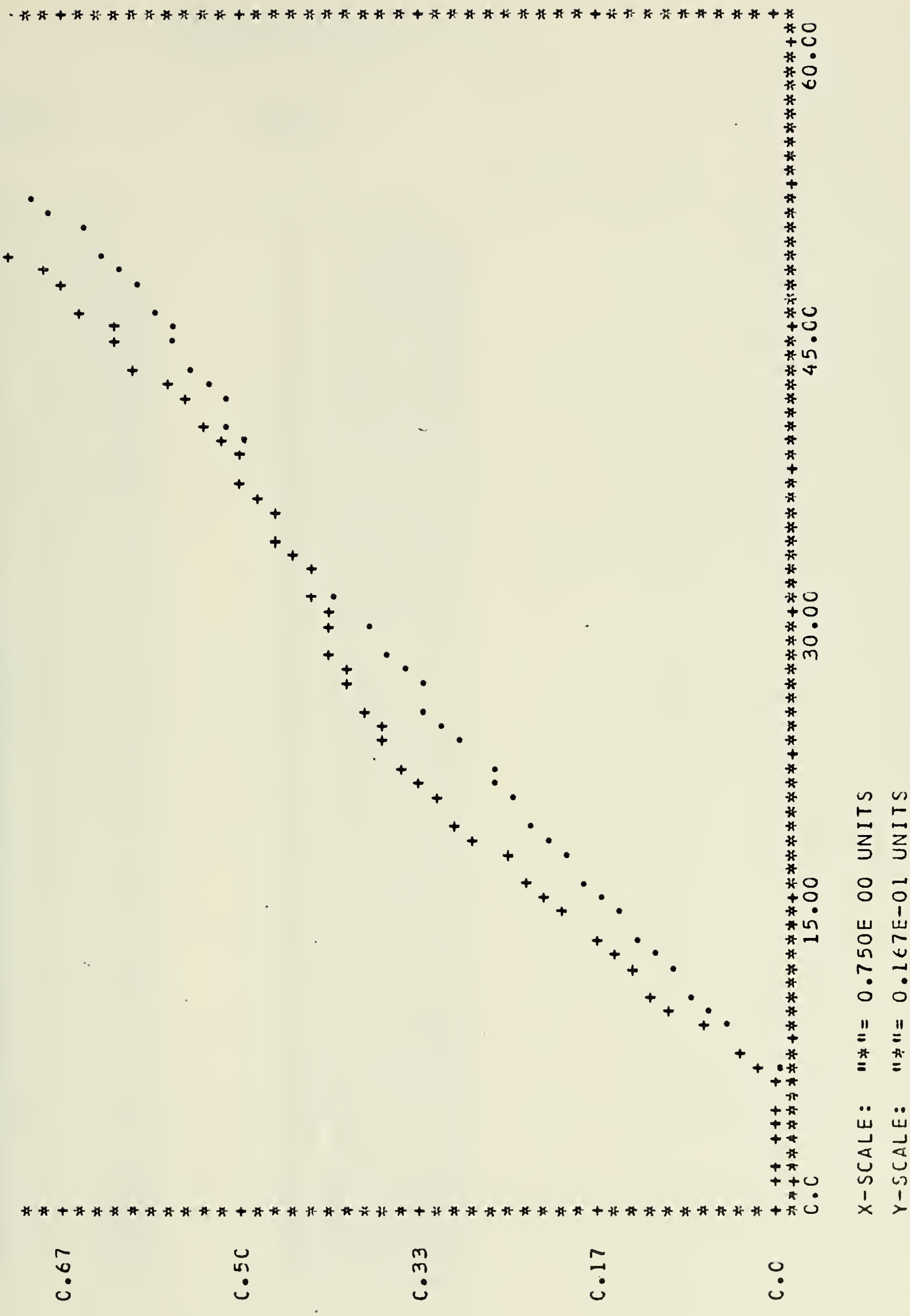
X-SCALE: "*"= 0.75CE CO UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARAMETERS
SEARCHER SPEED=14.0
ASSUMED TARGET SPEED= 4.0
TIME LATE=12.0

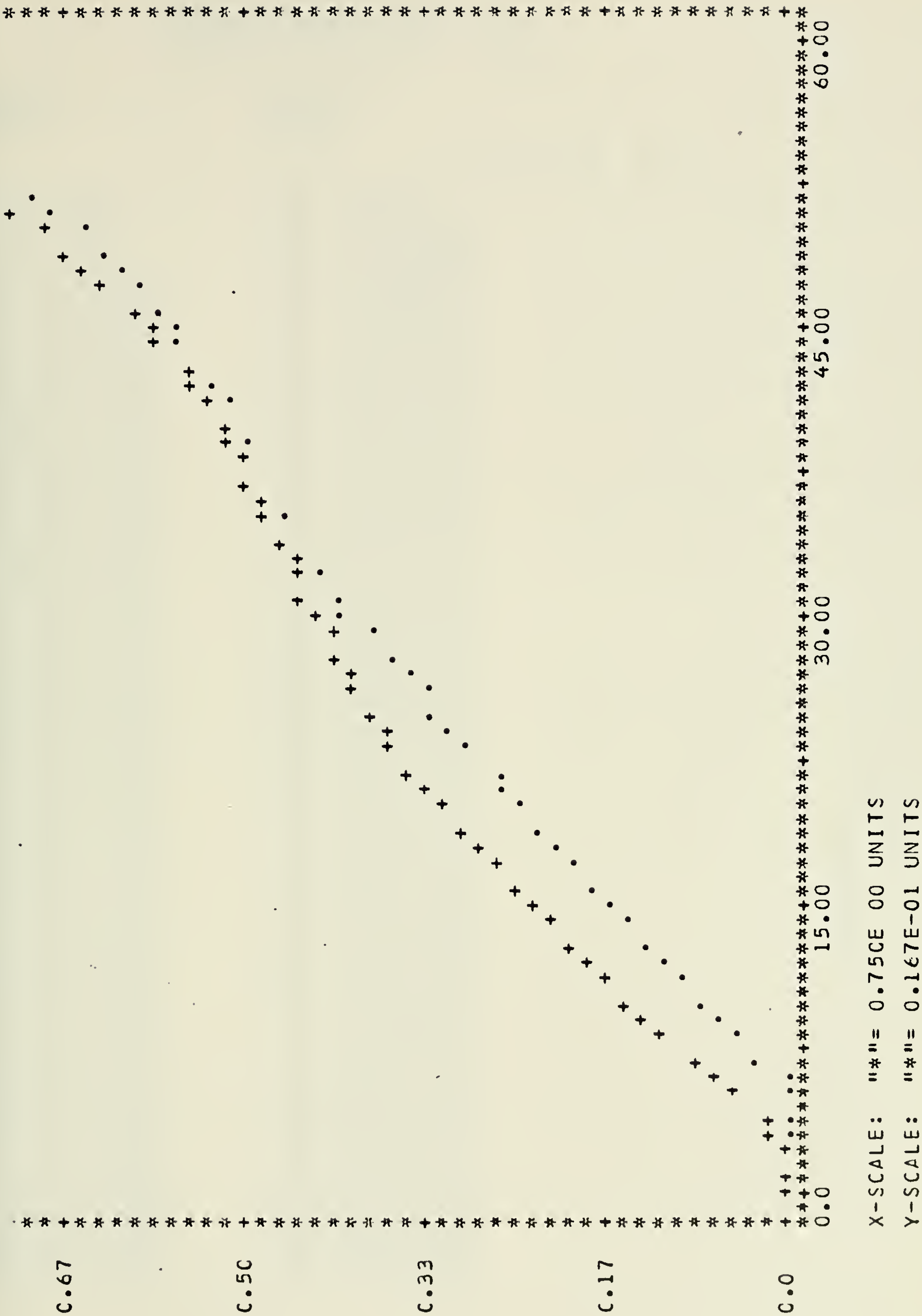
```
SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME=   96.00
```

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN
	FACTOR= 0.8 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0
CDF 3=0.0	CDF 3=0.0
CDF 4=0.0	CDF 4=0.0
CDF 5=0.0010	CDF 5=0.0
CDF 6=0.0030	CDF 6=0.0040
CDF 7=0.0060	CDF 7=0.0130
CDF 8=0.0250	CDF 8=0.0370
CDF 9=0.0450	CDF 9=0.0600
CDF 10=0.0670	CDF 10=0.0920
CDF 11=0.0870	CDF 11=0.1170
CDF 12=0.1050	CDF 12=0.1320
CDF 13=0.1200	CDF 13=0.1450
CDF 14=0.1380	CDF 14=0.1680
CDF 15=0.1550	CDF 15=0.1920
CDF 16=0.1690	CDF 16=0.2130
CDF 17=0.1880	CDF 17=0.2360
CDF 18=0.2020	CDF 18=0.2550
CDF 19=0.2140	CDF 19=0.2770
CDF 20=0.2290	CDF 20=0.2980
CDF 21=0.2440	CDF 21=0.3100
CDF 22=0.2590	CDF 22=0.3330
CDF 23=0.2740	CDF 23=0.3500
CDF 24=0.2920	CDF 24=0.3630
CDF 25=0.3120	CDF 25=0.3710
CDF 26=0.3270	CDF 26=0.3860
CDF 27=0.3410	CDF 27=0.3920
CDF 28=0.3560	CDF 28=0.3980
CDF 29=0.3720	CDF 29=0.4090
CDF 30=0.3910	CDF 30=0.4190
CDF 31=0.4090	CDF 31=0.4230
CDF 32=0.4240	CDF 32=0.4310
CDF 33=0.4390	CDF 33=0.4390
CDF 34=0.4480	CDF 34=0.4500
CDF 35=0.4600	CDF 35=0.4640
CDF 36=0.4730	CDF 36=0.4710
CDF 37=0.4800	CDF 37=0.4850
CDF 38=0.4920	CDF 38=0.4940
CDF 39=0.5010	CDF 39=0.5080
CDF 40=0.5060	CDF 40=0.5220
CDF 41=0.5170	CDF 41=0.5410
CDF 42=0.5220	CDF 42=0.5530
CDF 43=0.5340	CDF 43=0.5710
CDF 44=0.5480	CDF 44=0.5960
CDF 45=0.5600	CDF 45=0.6110
CDF 46=0.5680	CDF 46=0.6240
CDF 47=0.5790	CDF 47=0.6440
CDF 48=0.5930	CDF 48=0.6660
CDF 49=0.6110	CDF 49=0.6900
CDF 50=0.6300	CDF 50=0.7150
CDF 51=0.6520	CDF 51=0.7410
CDF 52=0.6760	CDF 52=0.7700
CDF 53=0.7040	CDF 53=0.7910
CDF 54=0.7270	CDF 54=0.8250
CDF 55=0.7600	CDF 55=0.8430
CDF 56=0.7940	CDF 56=0.8650
CDF 57=0.8490	CDF 57=0.8870
CDF 58=0.9100	CDF 58=0.9210
CDF 59=0.9630	CDF 59=0.9670
CDF 60=0.9920	CDF 60=0.9860
CDF 61=1.0000	CDF 61=1.0000

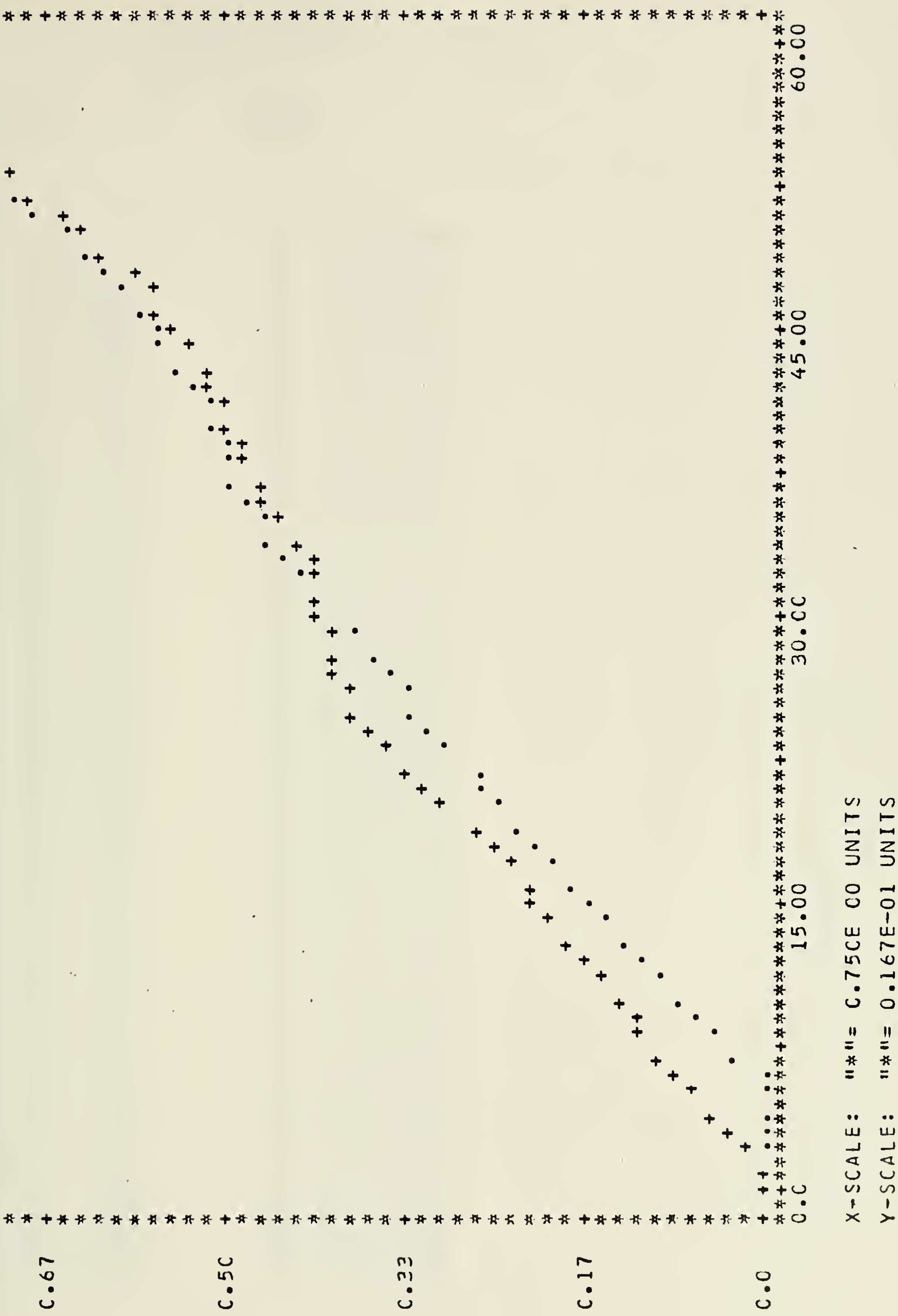

C.67
C.5C
C.33
C.17
C.O
C.C
15.00
30.00
45.CC
60.CO
X-SCALE: "*"= 0.750E 00 UNITS
Y-SCALE: "*"= 0.167E-01 UNITS

SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME= 96.00

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN FACTOR= 0.9 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0030
CDF 3=0.0	CDF 3=0.0050
CDF 4=0.0	CDF 4=0.0090
CDF 5=0.0010	CDF 5=0.0240
CDF 6=0.0030	CDF 6=0.0460
CDF 7=0.0060	CDF 7=0.0600
CDF 8=0.0250	CDF 8=0.0890
CDF 9=0.0450	CDF 9=0.1110
CDF 10=0.0670	CDF 10=0.1330
CDF 11=0.0870	CDF 11=0.1490
CDF 12=0.1050	CDF 12=0.1650
CDF 13=0.1200	CDF 13=0.1860
CDF 14=0.1380	CDF 14=0.2020
CDF 15=0.1550	CDF 15=0.2200
CDF 16=0.1690	CDF 16=0.2350
CDF 17=0.1880	CDF 17=0.2510
CDF 18=0.2020	CDF 18=0.2700
CDF 19=0.2140	CDF 19=0.2800
CDF 20=0.2290	CDF 20=0.2960
CDF 21=0.2440	CDF 21=0.3190
CDF 22=0.2590	CDF 22=0.3300
CDF 23=0.2740	CDF 23=0.3500
CDF 24=0.2920	CDF 24=0.3590
CDF 25=0.3120	CDF 25=0.3730
CDF 26=0.3270	CDF 26=0.3830
CDF 27=0.3410	CDF 27=0.3970
CDF 28=0.3560	CDF 28=0.4070
CDF 29=0.3720	CDF 29=0.4100
CDF 30=0.3910	CDF 30=0.4210
CDF 31=0.4090	CDF 31=0.4300
CDF 32=0.4240	CDF 32=0.4420
CDF 33=0.4390	CDF 33=0.4470
CDF 34=0.4480	CDF 34=0.4530
CDF 35=0.4600	CDF 35=0.4650
CDF 36=0.4730	CDF 36=0.4810
CDF 37=0.4800	CDF 37=0.4860
CDF 38=0.4920	CDF 38=0.4970
CDF 39=0.5010	CDF 39=0.5070
CDF 40=0.5060	CDF 40=0.5150
CDF 41=0.5170	CDF 41=0.5250
CDF 42=0.5220	CDF 42=0.5400
CDF 43=0.5340	CDF 43=0.5470
CDF 44=0.5480	CDF 44=0.5540
CDF 45=0.5600	CDF 45=0.5800
CDF 46=0.5680	CDF 46=0.5880
CDF 47=0.5790	CDF 47=0.6070
CDF 48=0.5930	CDF 48=0.6280
CDF 49=0.6110	CDF 49=0.6450
CDF 50=0.6300	CDF 50=0.6670
CDF 51=0.6520	CDF 51=0.6890
CDF 52=0.6760	CDF 52=0.7100
CDF 53=0.7040	CDF 53=0.7360
CDF 54=0.7270	CDF 54=0.7560
CDF 55=0.7600	CDF 55=0.7810
CDF 56=0.7940	CDF 56=0.7990
CDF 57=0.8490	CDF 57=0.8190
CDF 58=0.9100	CDF 58=0.8370
CDF 59=0.9630	CDF 59=0.8590
CDF 60=0.9920	CDF 60=0.8810
CDF 61=1.0000	CDF 61=1.0000

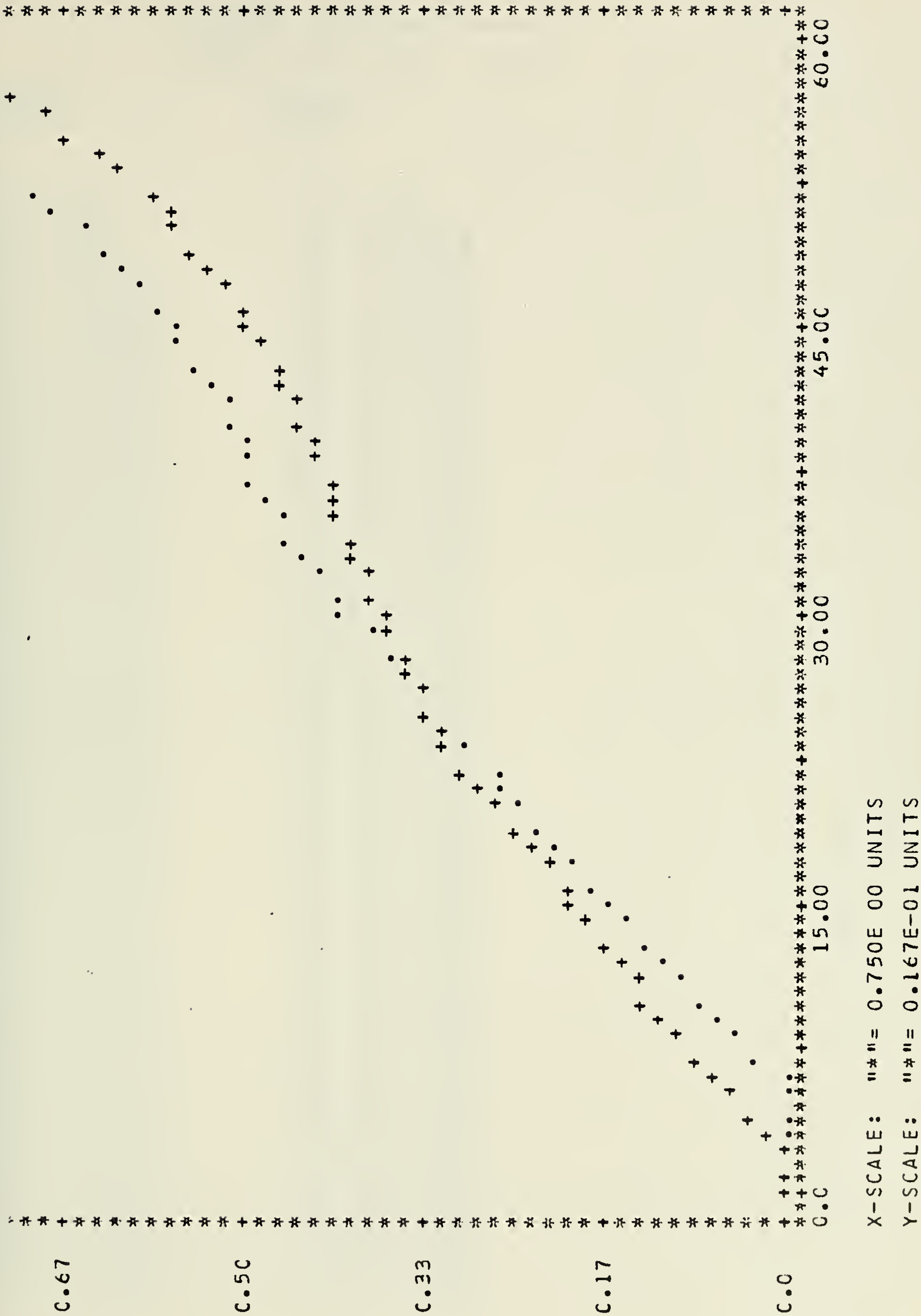

C.67
C.5C
C.33
C.17
C.0
0.0
15.00
30.00
45.00
60.00
X-SCALE: "*"= 0.75CE 00 UNITS
Y-SCALE: "*"= 0.167E-01 UNITS

```
                    SEARCHER SPEED=12.0
                    ASSUMED TARGET SPEED= 8.0
                    TIME LATE= 4.0
                    TOTAL SEARCH TIME=    96.00

KOOPMAN                      ALTERNATE
SEARCH                       SEARCH
PLAN                         PLAN

                             FACTOR= 1.0  MULTIPLIER=  2.00

 CDF  1=0.0                   CDF  1=0.0
 CDF  2=0.0                   CDF  2=0.0080
 CDF  3=0.0                   CDF  3=0.0190
 CDF  4=0.0                   CDF  4=0.0330
 CDF  5=0.0010                CDF  5=0.0470
 CDF  6=0.0030                CDF  6=0.0600
 CDF  7=0.0060                CDF  7=0.0810
 CDF  8=0.0250                CDF  8=0.1000
 CDF  9=0.0450                CDF  9=0.1090
 CDF 10=0.0670                CDF 10=0.1230
 CDF 11=0.0870                CDF 11=0.1360
 CDF 12=0.1050                CDF 12=0.1560
 CDF 13=0.1200                CDF 13=0.1660
 CDF 14=0.1380                CDF 14=0.1810
 CDF 15=0.1550                CDF 15=0.1940
 CDF 16=0.1690                CDF 16=0.2100
 CDF 17=0.1880                CDF 17=0.2230
 CDF 18=0.2020                CDF 18=0.2330
 CDF 19=0.2140                CDF 19=0.2510
 CDF 20=0.2290                CDF 20=0.2740
 CDF 21=0.2440                CDF 21=0.2920
 CDF 22=0.2590                CDF 22=0.3140
 CDF 23=0.2740                CDF 23=0.3300
 CDF 24=0.2920                CDF 24=0.3470
 CDF 25=0.3120                CDF 25=0.3620
 CDF 26=0.3270                CDF 26=0.3760
 CDF 27=0.3410                CDF 27=0.3870
 CDF 28=0.3560                CDF 28=0.3960
 CDF 29=0.3720                CDF 29=0.3980
 CDF 30=0.3910                CDF 30=0.4060
 CDF 31=0.4090                CDF 31=0.4110
 CDF 32=0.4240                CDF 32=0.4170
 CDF 33=0.4390                CDF 33=0.4220
 CDF 34=0.4480                CDF 34=0.4250
 CDF 35=0.4600                CDF 35=0.4370
 CDF 36=0.4730                CDF 36=0.4500
 CDF 37=0.4800                CDF 37=0.4600
 CDF 38=0.4920                CDF 38=0.4700
 CDF 39=0.5010                CDF 39=0.4790
 CDF 40=0.5060                CDF 40=0.4890
 CDF 41=0.5170                CDF 41=0.4960
 CDF 42=0.5220                CDF 42=0.5040
 CDF 43=0.5340                CDF 43=0.5130
 CDF 44=0.5480                CDF 44=0.5240
 CDF 45=0.5600                CDF 45=0.5330
 CDF 46=0.5680                CDF 46=0.5470
 CDF 47=0.5790                CDF 47=0.5600
 CDF 48=0.5930                CDF 48=0.5730
 CDF 49=0.6110                CDF 49=0.5900
 CDF 50=0.6300                CDF 50=0.6190
 CDF 51=0.6520                CDF 51=0.6410
 CDF 52=0.6760                CDF 52=0.6550
 CDF 53=0.7040                CDF 53=0.6770
 CDF 54=0.7270                CDF 54=0.6930
 CDF 55=0.7600                CDF 55=0.7110
 CDF 56=0.7940                CDF 56=0.7440
 CDF 57=0.8490                CDF 57=0.7560
 CDF 58=0.9100                CDF 58=0.7800
 CDF 59=0.9630                CDF 59=0.8020
 CDF 60=0.9920                CDF 60=0.8190
 CDF 61=1.0000                CDF 61=1.0000
```


C.67
C.5C
C.33
C.17
C.0
0.C
15.00
30.CC
45.00
60.00
X-SCALE: "*"= C.75CE CO UNITS
Y-SCALE: "*"= 0.167E-01 UNITS

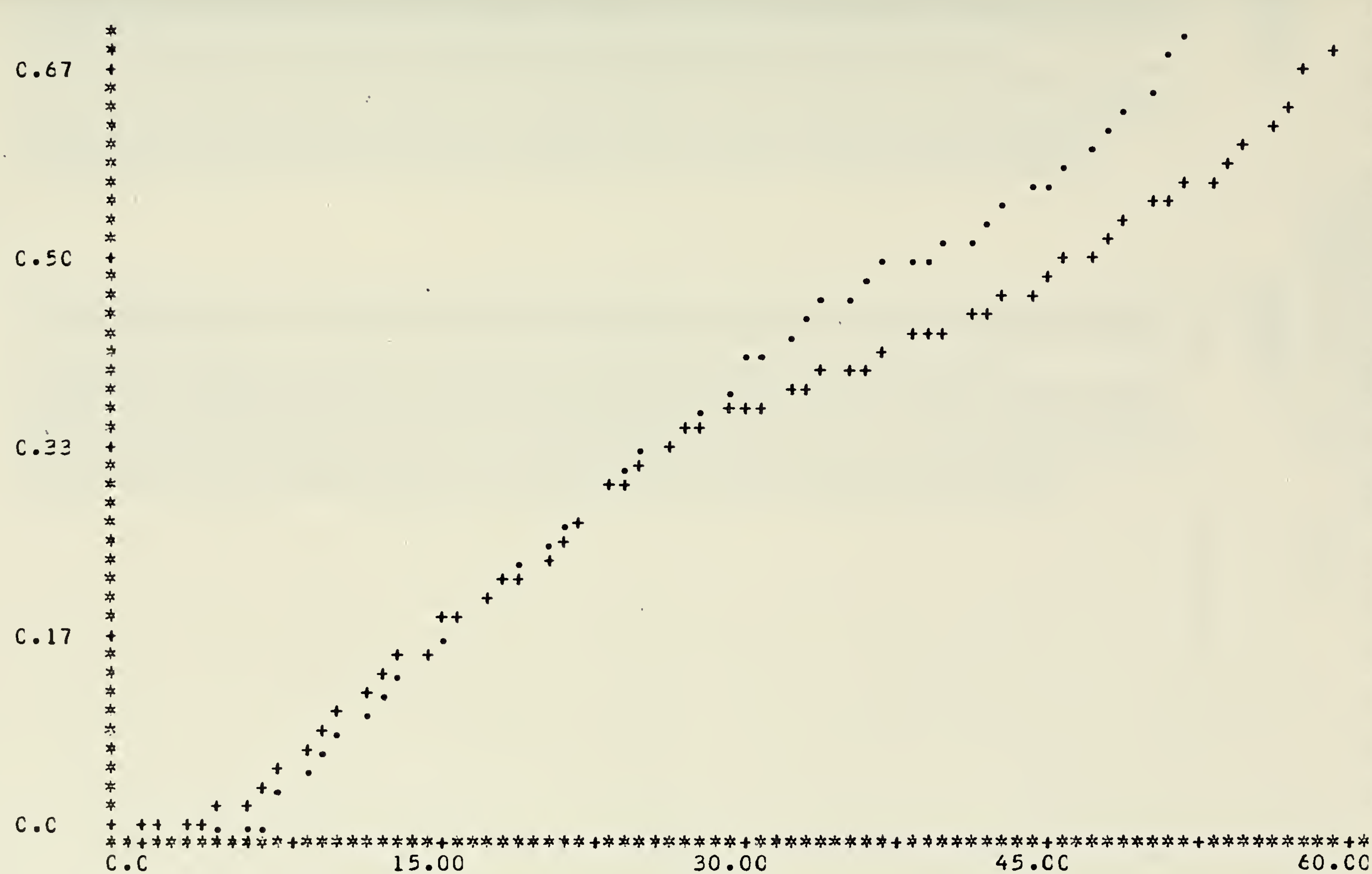
SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME= 96.00

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN
	FACTOR= 1.1 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0020
CDF 3=0.0	CDF 3=0.0070
CDF 4=0.0	CDF 4=0.0140
CDF 5=0.0010	CDF 5=0.0340
CDF 6=0.0030	CDF 6=0.0540
CDF 7=0.0060	CDF 7=0.0700
CDF 8=0.0250	CDF 8=0.0880
CDF 9=0.0450	CDF 9=0.0990
CDF 10=0.0670	CDF 10=0.1130
CDF 11=0.0870	CDF 11=0.1250
CDF 12=0.1050	CDF 12=0.1370
CDF 13=0.1200	CDF 13=0.1500
CDF 14=0.1380	CDF 14=0.1630
CDF 15=0.1550	CDF 15=0.1850
CDF 16=0.1690	CDF 16=0.1960
CDF 17=0.1880	CDF 17=0.2060
CDF 18=0.2020	CDF 18=0.2170
CDF 19=0.2140	CDF 19=0.2350
CDF 20=0.2290	CDF 20=0.2470
CDF 21=0.2440	CDF 21=0.2670
CDF 22=0.2590	CDF 22=0.2780
CDF 23=0.2740	CDF 23=0.2970
CDF 24=0.2920	CDF 24=0.3100
CDF 25=0.3120	CDF 25=0.3230
CDF 26=0.3270	CDF 26=0.3300
CDF 27=0.3410	CDF 27=0.3390
CDF 28=0.3560	CDF 28=0.3490
CDF 29=0.3720	CDF 29=0.3570
CDF 30=0.3910	CDF 30=0.3660
CDF 31=0.4090	CDF 31=0.3750
CDF 32=0.4240	CDF 32=0.3820
CDF 33=0.4390	CDF 33=0.3900
CDF 34=0.4480	CDF 34=0.3980
CDF 35=0.4600	CDF 35=0.4060
CDF 36=0.4730	CDF 36=0.4120
CDF 37=0.4800	CDF 37=0.4140
CDF 38=0.4920	CDF 38=0.4170
CDF 39=0.5010	CDF 39=0.4280
CDF 40=0.5060	CDF 40=0.4380
CDF 41=0.5170	CDF 41=0.4480
CDF 42=0.5220	CDF 42=0.4550
CDF 43=0.5340	CDF 43=0.4620
CDF 44=0.5480	CDF 44=0.4700
CDF 45=0.5600	CDF 45=0.4840
CDF 46=0.5680	CDF 46=0.4950
CDF 47=0.5790	CDF 47=0.5030
CDF 48=0.5930	CDF 48=0.5230
CDF 49=0.6110	CDF 49=0.5300
CDF 50=0.6300	CDF 50=0.5470
CDF 51=0.6520	CDF 51=0.5650
CDF 52=0.6760	CDF 52=0.5750
CDF 53=0.7040	CDF 53=0.5880
CDF 54=0.7270	CDF 54=0.6170
CDF 55=0.7600	CDF 55=0.6370
CDF 56=0.7940	CDF 56=0.6620
CDF 57=0.8490	CDF 57=0.6860
CDF 58=0.9100	CDF 58=0.7110
CDF 59=0.9630	CDF 59=0.7290
CDF 60=0.9920	CDF 60=0.7590
CDF 61=1.0000	CDF 61=1.0000


C.67
C.5C
C.33
C.17
C.C
C.C
15.00
30.00
45.0C
60.CO
X-SCALE: "*"= 0.750E 00 UNITS
Y-SCALE: "*"= 0.167E-01 UNITS

```
SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME=   96.00
```

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN
	FACTOR= 1.2 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0
CDF 3=0.0	CDF 3=0.0020
CDF 4=0.0	CDF 4=0.0040
CDF 5=0.0010	CDF 5=0.0100
CDF 6=0.0030	CDF 6=0.0180
CDF 7=0.0060	CDF 7=0.0290
CDF 8=0.0250	CDF 8=0.0480
CDF 9=0.0450	CDF 9=0.0680
CDF 10=0.0670	CDF 10=0.0860
CDF 11=0.0870	CDF 11=0.0930
CDF 12=0.1050	CDF 12=0.1130
CDF 13=0.1200	CDF 13=0.1320
CDF 14=0.1380	CDF 14=0.1470
CDF 15=0.1550	CDF 15=0.1580
CDF 16=0.1690	CDF 16=0.1750
CDF 17=0.1880	CDF 17=0.1860
CDF 18=0.2020	CDF 18=0.2000
CDF 19=0.2140	CDF 19=0.2100
CDF 20=0.2290	CDF 20=0.2210
CDF 21=0.2440	CDF 21=0.2340
CDF 22=0.2590	CDF 22=0.2470
CDF 23=0.2740	CDF 23=0.2730
CDF 24=0.2920	CDF 24=0.2920
CDF 25=0.3120	CDF 25=0.3060
CDF 26=0.3270	CDF 26=0.3200
CDF 27=0.3410	CDF 27=0.3300
CDF 28=0.3560	CDF 28=0.3430
CDF 29=0.3720	CDF 29=0.3510
CDF 30=0.3910	CDF 30=0.3620
CDF 31=0.4090	CDF 31=0.3660
CDF 32=0.4240	CDF 32=0.3710
CDF 33=0.4390	CDF 33=0.3780
CDF 34=0.4480	CDF 34=0.3860
CDF 35=0.4600	CDF 35=0.3920
CDF 36=0.4730	CDF 36=0.3970
CDF 37=0.4800	CDF 37=0.4080
CDF 38=0.4920	CDF 38=0.4180
CDF 39=0.5010	CDF 39=0.4250
CDF 40=0.5060	CDF 40=0.4320
CDF 41=0.5170	CDF 41=0.4380
CDF 42=0.5220	CDF 42=0.4490
CDF 43=0.5340	CDF 43=0.4540
CDF 44=0.5480	CDF 44=0.4620
CDF 45=0.5600	CDF 45=0.4720
CDF 46=0.5680	CDF 46=0.4800
CDF 47=0.5790	CDF 47=0.4920
CDF 48=0.5930	CDF 48=0.5000
CDF 49=0.6110	CDF 49=0.5130
CDF 50=0.6300	CDF 50=0.5330
CDF 51=0.6520	CDF 51=0.5420
CDF 52=0.6760	CDF 52=0.5550
CDF 53=0.7040	CDF 53=0.5640
CDF 54=0.7270	CDF 54=0.5740
CDF 55=0.7600	CDF 55=0.5840
CDF 56=0.7940	CDF 56=0.6010
CDF 57=0.8490	CDF 57=0.6180
CDF 58=0.9100	CDF 58=0.6390
CDF 59=0.9630	CDF 59=0.6590
CDF 60=0.9920	CDF 60=0.6800
CDF 61=1.0000	CDF 61=1.0000

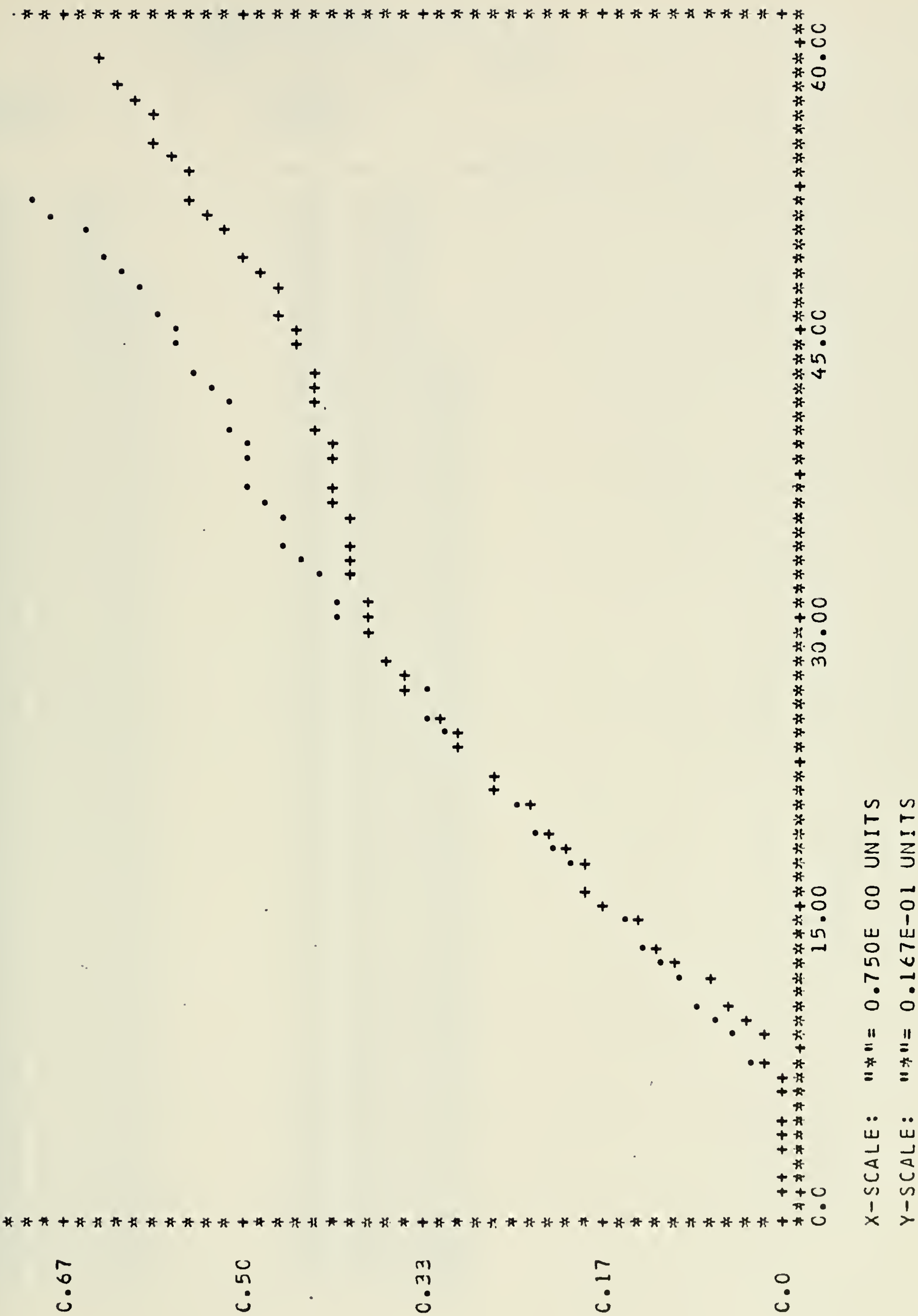


X-SCALE: "*"= 0.750E 00 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

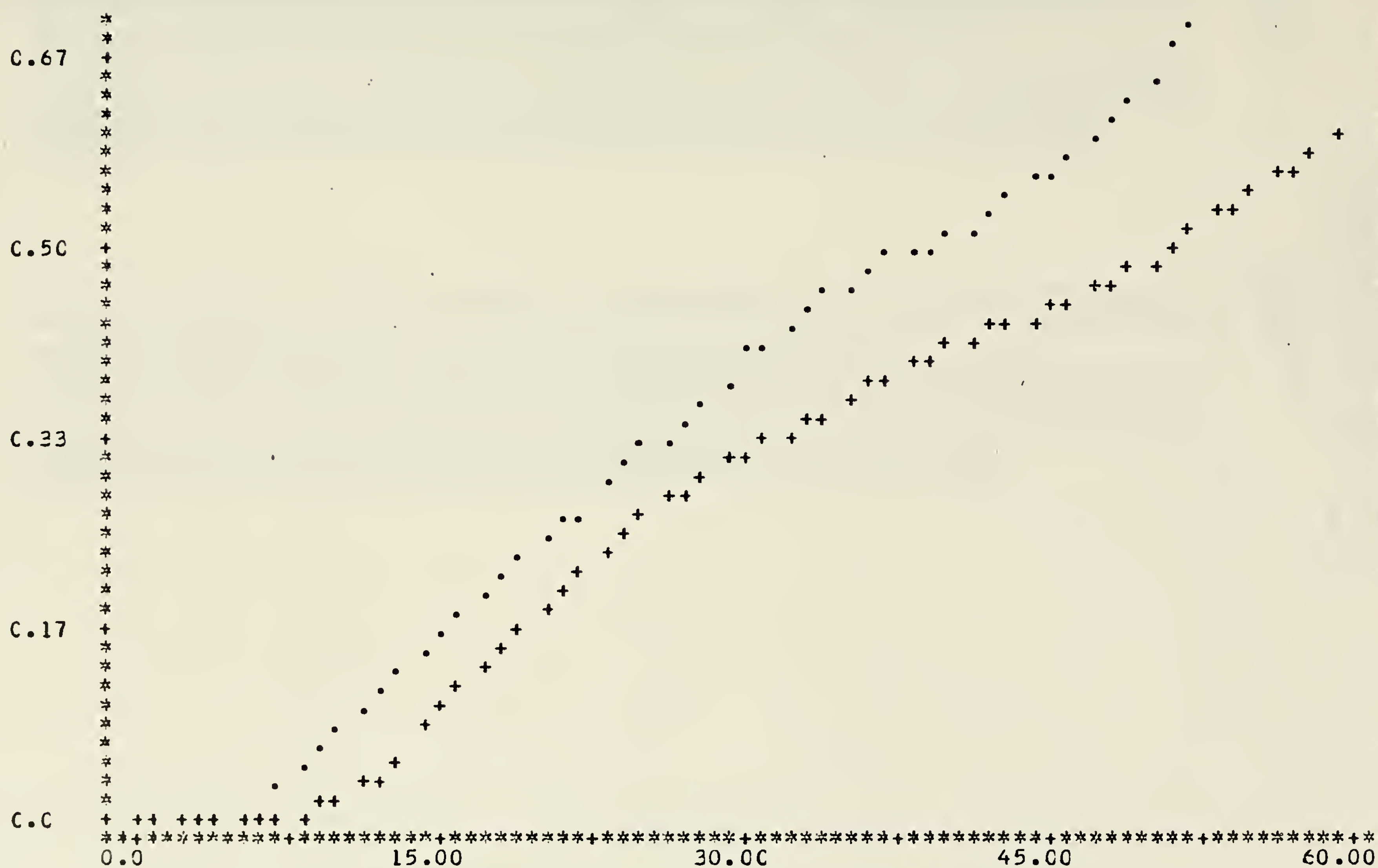
SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME= 96.00

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN
	FACTOR= 1.3 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0
CDF 3=0.0	CDF 3=0.0
CDF 4=0.0	CDF 4=0.0
CDF 5=0.0010	CDF 5=0.0
CDF 6=0.0030	CDF 6=0.0020
CDF 7=0.0060	CDF 7=0.0060
CDF 8=0.0250	CDF 8=0.0150
CDF 9=0.0450	CDF 9=0.0240
CDF 10=0.0670	CDF 10=0.0300
CDF 11=0.0870	CDF 11=0.0420
CDF 12=0.1050	CDF 12=0.0690
CDF 13=0.1200	CDF 13=0.0970
CDF 14=0.1380	CDF 14=0.1120
CDF 15=0.1550	CDF 15=0.1400
CDF 16=0.1690	CDF 16=0.1610
CDF 17=0.1880	CDF 17=0.1760
CDF 18=0.2020	CDF 18=0.1850
CDF 19=0.2140	CDF 19=0.1960
CDF 20=0.2290	CDF 20=0.2140
CDF 21=0.2440	CDF 21=0.2310
CDF 22=0.2590	CDF 22=0.2590
CDF 23=0.2740	CDF 23=0.2710
CDF 24=0.2920	CDF 24=0.2920
CDF 25=0.3120	CDF 25=0.3070
CDF 26=0.3270	CDF 26=0.3220
CDF 27=0.3410	CDF 27=0.3420
CDF 28=0.3560	CDF 28=0.3530
CDF 29=0.3720	CDF 29=0.3670
CDF 30=0.3910	CDF 30=0.3760
CDF 31=0.4090	CDF 31=0.3820
CDF 32=0.4240	CDF 32=0.3860
CDF 33=0.4390	CDF 33=0.3960
CDF 34=0.4480	CDF 34=0.4000
CDF 35=0.4600	CDF 35=0.4010
CDF 36=0.4730	CDF 36=0.4040
CDF 37=0.4800	CDF 37=0.4110
CDF 38=0.4920	CDF 38=0.4180
CDF 39=0.5010	CDF 39=0.4210
CDF 40=0.5060	CDF 40=0.4230
CDF 41=0.5170	CDF 41=0.4270
CDF 42=0.5220	CDF 42=0.4280
CDF 43=0.5340	CDF 43=0.4340
CDF 44=0.5480	CDF 44=0.4400
CDF 45=0.5600	CDF 45=0.4490
CDF 46=0.5680	CDF 46=0.4570
CDF 47=0.5790	CDF 47=0.4650
CDF 48=0.5930	CDF 48=0.4730
CDF 49=0.6110	CDF 49=0.4850
CDF 50=0.6300	CDF 50=0.4990
CDF 51=0.6520	CDF 51=0.5150
CDF 52=0.6760	CDF 52=0.5280
CDF 53=0.7040	CDF 53=0.5420
CDF 54=0.7270	CDF 54=0.5560
CDF 55=0.7600	CDF 55=0.5690
CDF 56=0.7940	CDF 56=0.5760
CDF 57=0.8490	CDF 57=0.5890
CDF 58=0.9100	CDF 58=0.5940
CDF 59=0.9630	CDF 59=0.6180
CDF 60=0.9920	CDF 60=0.6330
CDF 61=1.0000	CDF 61=1.0000


C.67
C.5C
C.33
C.17
C.0
C.C
15.00
30.00
45.CC
60.CC
X-SCALE: "*"= 0.750E 00 UNITS
Y-SCALE: "*"= 0.167E-01 UNITS

SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME= 96.00

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN FACTOR= 1.4 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0
CDF 3=0.0	CDF 3=0.0
CDF 4=0.0	CDF 4=0.0
CDF 5=0.0010	CDF 5=0.0
CDF 6=0.0030	CDF 6=0.0
CDF 7=0.0060	CDF 7=0.0
CDF 8=0.0250	CDF 8=0.0
CDF 9=0.0450	CDF 9=0.0010
CDF 10=0.0670	CDF 10=0.0140
CDF 11=0.0870	CDF 11=0.0230
CDF 12=0.1050	CDF 12=0.0290
CDF 13=0.1200	CDF 13=0.0410
CDF 14=0.1380	CDF 14=0.0560
CDF 15=0.1550	CDF 15=0.0800
CDF 16=0.1690	CDF 16=0.1040
CDF 17=0.1880	CDF 17=0.1220
CDF 18=0.2020	CDF 18=0.1360
CDF 19=0.2140	CDF 19=0.1550
CDF 20=0.2290	CDF 20=0.1690
CDF 21=0.2440	CDF 21=0.1840
CDF 22=0.2590	CDF 22=0.1960
CDF 23=0.2740	CDF 23=0.2150
CDF 24=0.2920	CDF 24=0.2340
CDF 25=0.3120	CDF 25=0.2530
CDF 26=0.3270	CDF 26=0.2650
CDF 27=0.3410	CDF 27=0.2780
CDF 28=0.3560	CDF 28=0.2880
CDF 29=0.3720	CDF 29=0.3010
CDF 30=0.3910	CDF 30=0.3110
CDF 31=0.4090	CDF 31=0.3230
CDF 32=0.4240	CDF 32=0.3330
CDF 33=0.4390	CDF 33=0.3410
CDF 34=0.4480	CDF 34=0.3460
CDF 35=0.4600	CDF 35=0.3580
CDF 36=0.4730	CDF 36=0.3670
CDF 37=0.4800	CDF 37=0.3760
CDF 38=0.4920	CDF 38=0.3870
CDF 39=0.5010	CDF 39=0.3950
CDF 40=0.5060	CDF 40=0.4060
CDF 41=0.5170	CDF 41=0.4140
CDF 42=0.5220	CDF 42=0.4190
CDF 43=0.5340	CDF 43=0.4250
CDF 44=0.5480	CDF 44=0.4320
CDF 45=0.5600	CDF 45=0.4400
CDF 46=0.5680	CDF 46=0.4460
CDF 47=0.5790	CDF 47=0.4530
CDF 48=0.5930	CDF 48=0.4600
CDF 49=0.6110	CDF 49=0.4710
CDF 50=0.6300	CDF 50=0.4790
CDF 51=0.6520	CDF 51=0.4860
CDF 52=0.6760	CDF 52=0.5050
CDF 53=0.7040	CDF 53=0.5150
CDF 54=0.7270	CDF 54=0.5270
CDF 55=0.7600	CDF 55=0.5390
CDF 56=0.7940	CDF 56=0.5490
CDF 57=0.8490	CDF 57=0.5620
CDF 58=0.9100	CDF 58=0.5710
CDF 59=0.9630	CDF 59=0.5880
CDF 60=0.9920	CDF 60=0.6020
CDF 61=1.0000	CDF 61=1.0000

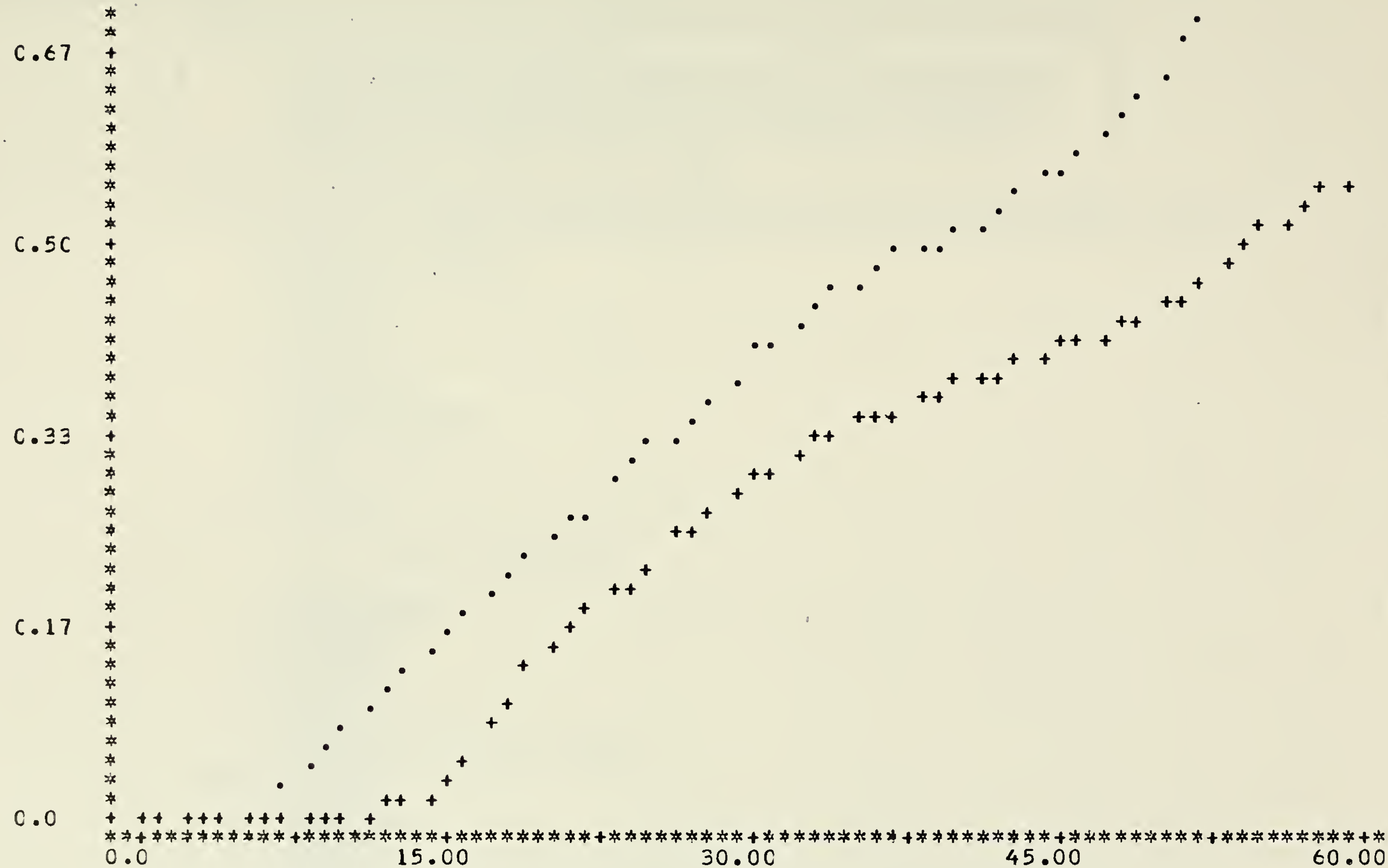


X-SCALE: "*"= 0.75CE C0 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

SEARCHER SPEED=12.0
ASSUMED TARGET SPEED= 8.0
TIME LATE= 4.0
TOTAL SEARCH TIME= 96.00

KOOPMAN SEARCH PLAN	ALTERNATE SEARCH PLAN FACTOR= 1.5 MULTIPLIER= 2.00
CDF 1=0.0	CDF 1=0.0
CDF 2=0.0	CDF 2=0.0
CDF 3=0.0	CDF 3=0.0
CDF 4=0.0	CDF 4=0.0
CDF 5=0.0010	CDF 5=0.0
CDF 6=0.0030	CDF 6=0.0
CDF 7=0.0060	CDF 7=0.0
CDF 8=0.0250	CDF 8=0.0
CDF 9=0.0450	CDF 9=0.0
CDF 10=0.0670	CDF 10=0.0010
CDF 11=0.0870	CDF 11=0.0010
CDF 12=0.1050	CDF 12=0.0040
CDF 13=0.1200	CDF 13=0.0100
CDF 14=0.1380	CDF 14=0.0170
CDF 15=0.1550	CDF 15=0.0230
CDF 16=0.1690	CDF 16=0.0350
CDF 17=0.1880	CDF 17=0.0510
CDF 18=0.2020	CDF 18=0.0770
CDF 19=0.2140	CDF 19=0.1040
CDF 20=0.2290	CDF 20=0.1310
CDF 21=0.2440	CDF 21=0.1460
CDF 22=0.2590	CDF 22=0.1610
CDF 23=0.2740	CDF 23=0.1770
CDF 24=0.2920	CDF 24=0.1930
CDF 25=0.3120	CDF 25=0.2050
CDF 26=0.3270	CDF 26=0.2230
CDF 27=0.3410	CDF 27=0.2440
CDF 28=0.3560	CDF 28=0.2580
CDF 29=0.3720	CDF 29=0.2710
CDF 30=0.3910	CDF 30=0.2880
CDF 31=0.4090	CDF 31=0.2970
CDF 32=0.4240	CDF 32=0.3060
CDF 33=0.4390	CDF 33=0.3120
CDF 34=0.4480	CDF 34=0.3270
CDF 35=0.4600	CDF 35=0.3360
CDF 36=0.4730	CDF 36=0.3430
CDF 37=0.4800	CDF 37=0.3480
CDF 38=0.4920	CDF 38=0.3530
CDF 39=0.5010	CDF 39=0.3640
CDF 40=0.5060	CDF 40=0.3710
CDF 41=0.5170	CDF 41=0.3790
CDF 42=0.5220	CDF 42=0.3820
CDF 43=0.5340	CDF 43=0.3870
CDF 44=0.5480	CDF 44=0.3950
CDF 45=0.5600	CDF 45=0.4020
CDF 46=0.5680	CDF 46=0.4110
CDF 47=0.5790	CDF 47=0.4190
CDF 48=0.5930	CDF 48=0.4250
CDF 49=0.6110	CDF 49=0.4320
CDF 50=0.6300	CDF 50=0.4410
CDF 51=0.6520	CDF 51=0.4490
CDF 52=0.6760	CDF 52=0.4570
CDF 53=0.7040	CDF 53=0.4650
CDF 54=0.7270	CDF 54=0.4800
CDF 55=0.7600	CDF 55=0.4990
CDF 56=0.7940	CDF 56=0.5130
CDF 57=0.8490	CDF 57=0.5210
CDF 58=0.9100	CDF 58=0.5310
CDF 59=0.9630	CDF 59=0.5440
CDF 60=0.9920	CDF 60=0.5530
CDF 61=1.0000	CDF 61=1.0000



X-SCALE: "*"= 0.75CE 00 UNITS

Y-SCALE: "*"= 0.167E-01 UNITS

```
C*********************************************************************
C*                                                                   *
C*     THIS PROGRAM CALCULATES ONE CIRCLIT OF THE KOCPMAN            *
C*     SEARCH PLAN FOR VARIOUS COMBINATICNS OF TARGET SPEEC          *
C*     SEARCHER SPEED AND TIME LATE. IT OLTPUTS A PRINTED            *
C*     LISTING OF THE PROBABILITY OF TARGET DETECTION AS A           *
C*     FUNCTION OF ACTUAL SONAR RANGE. THIS DATA IS ALSO             *
C*     PRESENTED IN A GRAPHICAL FORMAT                               *
C*                                                                   *
C*********************************************************************
      DIMENSION R(5),SL(4),TCC(5),XC(5),YC(5),A1(4),A2(4),A3(4),A4(4),
     1X(61),NTALLY(60),CDF(61),RSQ(9)
      DATA NTALLY/60*0/
      NRUNS=0
      NTRUN=3000
      STE=4.0
      ULCT=359.0 * 0.01745329
      LLCT=0.0
      BCXES=60.0
      RANGE=60.0
      DX=BOXES/RANGE
      X(1)=0.0
      DC 10 I=2,61
      II=I-1
      X(I)=FLOAT(II)
   10 CCNTINUE
C
C     RANDCM NUMBER GENERATOR
C
      KR=16807
      IRS=27496385
      IC=IRS *KR
   40 CCNTINUE
      ULST=STE
      LLST=STE
      TL=3.0
   50 CCNTINUE
      SC=8.0
      IF(STE.GE. SO) GO TO 700
  100 CCNTINUE
      MU=0.0
      SIGMA=2.0
      RS=10.0
      CRXC=0.0
      ORYC=0.0
C
C     CCMPUTE PARAMETERS NECESSARY TO CBTAIN A SEARCH PLAN
C
```

```
      E=RS/0.69315
      S=0.75 * SQRT( E * SIGMA)
      SM=(SO+STE)/(SO-STE)
      RATIO=SIGMA/STE
      IF(RATIO .LE. TL) GO TO 200
      R(1)=S/(SM**2+1.0)
      GC TO 220
  200 R(1)=0.8*STE*TL
  220 A=(SO*S)/(SO-STE)
C
C     CALCULATE R(2) THRU R(5)
C
      R(2)=SM * R(1)
      R(3)=SM * R(2)
      R(4)=SM * R(3)
      R(5)=SM * R(4) +A
C
C     CCMPUTE  SEARCH LEG LENGTHS
C
      SL(1)=SM * R(1)
      SL(2)= SM * SL(1) +R(1)
      SL(3)=SL(2) *SM
      SL(4)=SM*SL(3)+A
C
C     CCMPUTE TOTAL TIME SEARCHING
C
      TSL=SL(1)+SL(2) +SL(3)+SL(4)
      TST=TSL/SO
C
C     CALCULATE X AND Y CCORDINATES OF THE  SEARCH LEGS
C
      XC(1)=ORXC -R(1)
      YC(1)=ORYC
      XC(2)=XC(1)
      YC(2)=YC(1)+SL(1)
      XC(3)=XC(2)+SL(2)
      YC(3)=YC(2)
      XC(4)=XC(3)
      YC(4)=YC(3)-SL(3)
      XC(5)=XC(4)-SL(4)
      YC(5)=YC(4)
C
C     CALCULATION OF TIMES AT WHICF SEARCHER REACFS SEARCH
C                         LEG  COORDINATES
C
      TCC(1)=TL
      TCC(2)=TL +SL(1)/SO
      TCC(3)=TCC(2) +SL(2)/SO
```

```
      TCC(4)=TCC(3)+ SL(3)/SO
      TCC(5)=TCC(4)+ SL(4)/SO
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG ONE
C
      A1(1)=XC(1)
      A2(1)=0.0
      A3(1)=YC(1)
      A4(1)=SO
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG TWO
C
      A1(2)=XC(2)
      A2(2)=SO
      A3(2)=YC(2)
      A4(2)=0.0
C
C     DETERMINE INTIAL VALUES FOR SEARCH LEG THREE
C
      A1(3)=XC(3)
      A2(3)=0.0
      A3(3)=YC(3)
      A4(3)=-SO
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG FOUR
C
      A1(4)=XC(4)
      A2(4)=-SO
      A3(4)=YC(4)
      A4(4)=0.0
C
C     SELECT  TGT COURSE UNIFORMLY BETWEEN UPPER AND LOWER LIMIT
C
  153 CONTINUE
      IC=IC*KR
      RN1=0.5 +FLOAT(IC) * 2.328306E-10
      CT=RN1*(ULCT-LLCT) +LLCT
C
C     GENERATE DIRECTION AND DISTANCE OF DATUM UNCERTAINITY
C
      IC=IC*KR
      RN2=0.5 +FLOAT(IC) *2.328306E-10
      DCU=RN2*2.0*3.14159
  155 Z=0.0
      DO 160 J=1,12
      IC=IC*KR
      Z=Z+ FLOAT(IC) *2.328306E-10
  160 CONTINUE
```

```
      RN3=SIGMA *Z +FLOAT(MU)
      DSDU= ABS(RN3)
C
C     GENERATE TARGET SPEED UNIFORMLY BETWEEN UPPER AND LOWER LIMITS
C
      IC=IC*KR
      RN4=0.5+FLOAT(IC)*2.328306E-10
      STA=RN4*(ULST-LLST)+LLST
C
C     CALCULATE TARGET X AND Y VELOCITY COMPONENTS
C
  250 VTX=STA*SIN(CT)
      VTY=STA*COS(CT)
C
C     CALCULATE INITIAL TARGET X AND Y COORDINATES
C
      TGTXC=ORXC + DSDU * COS(DDU)
      TGTYC=ORYC + DSDU * SIN (DDU)
C
C     DETERMINE CANDIDATES FOR MINIMUM FROM SEARCH LEG INTERVAL
C
      DO 300 K=1,4
      KK=(K)+1
      TIMERT=-((A1(K)-TGTXC)*(A2(K)-VTX)+(A3(K)-TGTYC)*(A4(K)-VTY) )/
     1( (A2(K)-VTX)**2 + (A4(K)-VTY)**2)
      IF(TIMERT .LE. 0.0) GO TO 310
      IF(TIMERT.GT. TCC(KK)) GO TO 310
      IF(TIMERT .LT. TCC(K)) GO TO 310
      SXC=A1(K)+A2(K)*TIMERT
      SYC=A3(K)+A4(K)*TIMERT
      TXC=VTX* TIMERT +TGTXC
      TYC=VTY * TIMERT + TGTYC
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
      GO TO 300
  310 RSQ(K)=9.9999E+10
  300 CONTINUE
C
C     DETERMINE END POINT CANDIDATES FOR MINIMUM
C
      DO 400 K=5,9
      K1=(K)-4
      TIME=TCC(K1)
      SXC=XC(K1)
      SYC=YC(K1)
      TXC=TGTXC +VTX*TIME
      TYC=TGTYC + VTY * TIME
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
  400 CONTINUE
```

```
C
C
C
C     DETERMINE THE CLOSEST POINT OF APPROACH FCR THIS ITERATION
C
      RSCM=RSQ(1)
      DC 500 K=2,9
      RMIN=AMIN1(RSQM,RSQ(K) )
      RSCM=RMIN
  5CC CCNTINUE
      RCPA=SQRT(RSQM)
C
C     TALLY CPA RANGES INTO BCXES
C
      IF(RCPA.GE.59.0) GO TO 610
      LR=DX* RCPA +1.0
      NTALLY(LR)=NTALLY(LR)+ 1
      GC TO 1150
  61C NTALLY(60)=NTALLY(60) +1
C
C     INCREMENT NUMBER OF RUNS
C
 1150 NRUNS=NRUNS +1
C
C     TEST TO SEE IF DESIRED NUMBER OF RUNS OBTAINED
C
      IF(NRUNS .EQ. NTRUN) GO TO 1200
      GC TO 153
 120C CCNTINUE
C
C     RESET ITERATION COUNTER TO ZERO
C
      NRUNS=0
C
C     CCMPUTE TOTAL NUMBER OF TALLIES
C
      NTSUM=0.0
      DC 650 L=1,60
      NTSUM= NTSUM + NTALLY(L)
  65C CONTINUE
C
C     CCMPUTE INDIVIDUAL RANGE PROBABILITIES ANC THE
C     CUMULATIVE PROBABILITIES
C
      CDF(1)=C.0
      SUMP=0.C
      DC 670 M=1,60
      MM=M+1
```

```
      PBCX=FLCAT(NTALLY(M))/FLOAT(NTSUM)
      SLMP=SUMP + PBOX
      CDF(MM)=SUMP
  67C CCNTINUE
      WRITE(6,9700) SO,STE,TL,TCC(5)
 9700 FCRMAT('1',  //////,25X,'KOOPMAN SEARCH PLAN',/,25X,'SEARCHER SPEE
     2D=',F4.1,/,25X,'ASSUMED TARGET SPEED=',F4.1,/,25X,'TIME LATE=',F4.
     31,/,25X,'TOTAL SEARCH TIME =',F1C.2,//)
      DC 1510 N=1,61
      WRITE(6,9520) N, CDF(N)
 9520 FORMAT(25X,'CDF',I4,'=',F7.5)
 151C CCNTINUE
      WRITE(6,9710)
 971C FCRMAT('1')
      CALL PLCTP(X,CDF,61,0)
      WRITE(6,9730)
 973C FCRMAT(//,10X,'SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARA
     1METERS')
      WRITE(6,9740) SO,STE,TL
 9740 FCRMAT(20X,'SEARCHER SPEED=',F4.1,/20X,'ASSUMED TARGET SPEED=',F4.
     11,/20X,'TIME LATE=',F4.1)
C
C
C     RESET ALL TALLY BOXES TC ZERO
C
      DC 690 I=1,60
      NTALLY(I)=0
  69C CCNTINUE
C     SLBROUTINE TO VARY SEARCHER SPEED
C
  7CC SC=SO+3.0
      IF(SO.LE.14.0)GO TC 100
      GC TO 710
C
C     SLBROUTINE TO CALCULATE TIME LATE INCREMENTS
C
  710 TL=TL* 2.0
      IF(TL.LE. 12.0) GO TO 50
      GC TO 720
C
C     SLBROUTINE TO CALCULATE TARGET SPEEDS DIFFERENT
C     FRCM ASSUMED SPEED USED IN CALCULATING SEARCH PLAN
C
  72C STE=STE +4.0
      IF(STE.LE.12.0) GO TO 40
      GC TO 2000
 2000 CCNTINUE
      STCP
```

END

```
****************************************************************
*                                                              *
*     THIS PROGRAM COMPARES THE KOOPMAN SEARCH PLAN FOR A      *
*     PARTICULAR TARGET SPEED, SEARCHER SPEED AND TIME LATE    *
*     AGAINST VARIOUS ALTERNATE SEARCH PLANS. THE AMOUNT OF    *
*     SEARCH TIME AVAILABLE IS SET AT A FIXED VALUE. EACH      *
*     SEARCH PLAN IS RUN FOR THIS FIXED AMOUNT OF TIME.ITS     *
*     OUTPUT IS A LISTING OF THE PROBABILITY OF TARGET         *
*     DETECTION AS A FUNCTION OF THE ACTUAL SONAR RANGE.THIS   *
*     INFORMATION IS ALSO PRESENTED IN A GRAPHICAL FORMAT      *
*                                                              *
****************************************************************
      DIMENSION X(61),R(5),SLK(20),XCK(21),YCK(21),A1(20),A2(20),A3(20),
     1A4(20),TCK(21),RSQ(150),XCO(101),YCO(101),B1(100),B2(100),B3(100),
     2B4(100),SLO(100),TCO(101),CDF(61),CDF2(61),NTALLY(60)
      DATA NTALLY/60*0/
C
C     INPUT CONSTANTS FOR THE MODEL
C
      STE=8.0
      TL=4.0
      SC=12.0
      TLIMIT=96.0
      NRUNS=0
      NTRUN=3000
      ULCT=359.0*0.0174539
      LLCT=0.0
      MU=0.0
      SIGMA=2.0
      ORXC=0.0
      ORYC=0.0
      RS=10.0
      BOXES=60.0
      RANGE=60.0
      DX=BOXES/RANGE
      X(1)=0.0
      DO 10 I=2,61
      II=I-1
      X(I)=FLOAT(II)
   10 CONTINUE
C
C     RANDOM NUMBER GENERATOR
C
      KR=16807
      IRS=27496385
      IC=KR*IRS
C
C     SET UPPER AND LOWER TARGET SPEED LIMITS
```

```
C
      ULST=STE
      LLST=STE
C
C     CCMPUTE PARAMETERS NECESSARY FOR A KOOPMAN SEARCH
C
      E=RS/0.69315
      S=C.75*SQRT(E*SIGMA)
      SM=(SO + STE)/(SO-STE)
      RATIO=SIGMA/STE
      IF(RATIC .LE.TL) GO TO 200
      R(1)= S/(SM**2 + 1.0)
      GC TO 220
  200 R(1)=0.8 * STE * TL
  22C A= (SO *S)/(SO-STE)
C
C     CCMPUTE KOCPMAN SEARCH LEG LENGTHS FOR FIVE CIRCUITS
C
      SLK(1)=SM* R(1)
      SLK(2)=SM*SLK(1)+R(1)
      SLK(3)=SM * SLK(2)
      SLK(4)=SM * SLK(3)+A
      SLK(5)=SM * SLK(4)
      SLK(6)=SM * SLK(5) +A
      SLK(7)=SM * SLK(6)
      SLK(8)=SM * SLK(7)- 2.0*A
      SLK(9)=SM * SLK(8)
      SLK(10)=SM * SLK(9)-2.0*A
      SLK(11)=SM * SLK(10)
      SLK(12)=SM * SLK(11) + 3.0*A
      SLK(13)=SM * SLK(12)
      SLK(14)=SM * SLK(13) + 3.0*A
      SLK(15)=SM * SLK(14)
      SLK(16)=SM * SLK(15) - 4.0 *A
      SLK(17)=SM * SLK(16)
      SLK(18)=SM * SLK(17) - 4.0 * A
      SLK(19)=SM * SLK(18)
      SLK(20)=SM * SLK(19) + 5.0 * A
C
C     CALCULATE X AND Y CCORDINATES OF THE SEARCH LEGS
C
      XCK(1)=CRXC-R(1)
      YCK(1)=CRYC
      XCK(2)=XCK(1)
      YCK(2)=YCK(1)+ SLK(1)
      XCK(3)=XCK(2)+ SLK(2)
      YCK(3)=YCK(2)
      XCK(4)=XCK(3)
```

```
      YCK(4)=YCK(3)- SLK(3)
      CC 230 KK=1,4
      KKC=4*(KK)
      KK1=4 *(KK)+1
      KK2=4* (KK)+2
      KK3=4*(KK)+3
      KK4=4*(KK)+4
      XCK(KK1)=XCK(KKO)-SLK(KKO)
      YCK(KK1)=YCK(KKO)
      XCK(KK2)=XCK(KK1)
      YCK(KK2)=YCK(KK1)+ SLK(KK1)
      XCK(KK3)=XCK(KK2)+ SLK(KK2)
      YCK(KK3)=YCK(KK2)
      XCK(KK4)=XCK(KK3)
      YCK(KK4)=YCK(KK3)- SLK(KK3)
  230 CCNTINUE
      XCK(21)=XCK(20)-SLK(20)
      YCK(21)=YCK(20)
C
C     CETERMINE INITIAL VALUES FOR SEARCH LEG CNE
C
      LV=1
  310 A1(LV)=XCK(LV)
      A2(LV)=C.C
      A3(LV)=YCK(LV)
      A4(LV)=SO
      LV=LV+4
      IF(LV.EC.21) GO TO 315
      GC TO 310
  315 CCNTINUE
C
C     CETERMINE INITIAL VALUES FOR SEARCH LEG TWC
C
      LV=2
  32C A1(LV)=XCK(LV)
      A2(LV)=SO
      A3(LV)=YCK(LV)
      A4(LV)=C.0
      LV=LV+4
      IF(LV.EC.22) GO TO 325
      GC TO 320
  325 CCNTINUE
C
C     CETERMINE INTIAL VALUES FOR SEARCH LEG THREE
C
      LV=3
  33C A1(LV)=XCK(LV)
      A2(LV)=C.0
```

```
      A3(LV)=YCK(LV)
      A4(LV)=-SO
      LV=LV+4
      IF(LV.EQ.23) GO TO 335
      GO TO 330
  335 CONTINUE
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG FOUR
C
      LV=4
  340 A1(LV)=XCK(LV)
      A2(LV)=-SO
      A2(LV)=-SO
      A3(LV)=YCK(LV)
      A4(LV)=C.0
      LV=LV+4
      IF(LV.EQ.24) GO TO 350
      GO TO 340
  350 CONTINUE
C
C     CALCULATION OF TIMES AT WHICH SEARCHER REACHES SEARCH
C                         LEG  COORDINATES
C
      TCK(1)=TL
      SLSUM=TL
      DO 360 II=2,21
      NSL=(II-1)
      SLSUM=SLSUM + SLK(NSL)/SO
      TCK(II)=SLSUM
      IF(TCK(II).GT.TLIMIT) GO TO 370
  360 CONTINUE
      NTC=NSL+1
  370 CONTINUE
C
C     SELECT  TGT COURSE UNIFORMLY BETWEEN UPPER AND LOWER LIMIT
C
  373 CONTINUE
      IC=IC*KR
      RN1=0.5 +FLOAT(IC) * 2.328306E-10
      CT=RN1*(ULCT-LLCT) +LLCT
C
C     GENERATE DIRECTION AND DISTANCE OF DATUM UNCERTAINITY
C
      IC=IC*KR
      RN2=0.5 +FLOAT(IC) *2.328306E-10
      DDU=RN2*2.0*3.14159
  375 Z=C.0
      DO 380 J=1,12
```

```
      IC=IC*KR
      Z=Z+ FLOAT(IC) *2.328306E-10
  380 CONTINUE
      RN3=SIGMA *Z +FLOAT(MU)
      DSDU= ABS(RN3)
C
C     GENERATE TARGET SPEED UNIFORMLY BETWEEN UPPER AND LOWER LIMITS
C
      IC=IC*KR
      RN4=0.5+FLOAT(IC)*2.328306E-10
      STA=RN4*(ULST-LLST)+LLST
C
C     CALCULATE INITIAL TARGET X AND Y COORDINATES
C
      TGTXC=ORXC + DSDU * COS(DDU)
      TGTYC=ORYC + DSDU * SIN (DDU)
C
C     CALCULATE TARGET X AND Y VELOCITY COMPONENTS
C
  385 VTX=STA* SIN(CT)
      VTY=STA*COS(CT)
C
C     DETERMINE CANDIDATES FOR MINIMUM FROM SEARCH LEG INTERVAL
C
      DO 400  K=1, NSL
      KK=(K)+1
      TIMERT=-((A1(K)-TGTXC)*(A2(K)-VTX)+(A3(K)-TGTYC)*(A4(K)-VTY) )/
     1(  (A2(K)-VTX)**2 +  (A4(K)-VTY)**2)
      IF(TIMERT.LE.0.0) GO TO 410
      IF(TIMERT.GT.TCK(KK)) GO TO 410
      IF(TIMERT.LT.TCK(K)) GO TO 410
      IF(TIMERT.GT.TLIMIT) GO TO 410
      SXC=A1(K)+A2(K)*TIMERT
      SYC=A3(K)+A4(K)*TIMERT
      TXC=VTX* TIMERT  +TGTXC
      TYC=VTY * TIMERT + TGTYC
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
      GO TO 400
  410 RSQ(K)=9.9999E+10
  400 CONTINUE
C
C     DETERMINE END POINT CANDIDATES FOR MINIMUM
C
      KPLUS1=NSL+1
      KPLUS2=2*NSL+1
      DO 450  K=KPLUS1, KPLUS2
      K1=(K)-NSL
      TIME=TCK(K1)
```

```
      IF(TIME.GE.TLIMIT) GO TO 460
      SXC=XCK(K1)
      SYC=YCK(K1)
      TXC=TGTXC  +VTX*TIME
      TYC=TGTYC  + VTY * TIME
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
  450 CONTINUE
      GO TO 470
  460 CONTINUE
      KPLUS2=(K)-1
  470 CONTINUE
C
C     DETERMINE THE CLOSEST POINT OF APPROACH FOR THIS ITERATION
C
      RSQM=RSQ(1)
      DO 500 K=2, KPLUS2
      RMIN=AMIN1(RSQM,RSQ(K) )
      RSQM=RMIN
  500 CONTINUE
      RCPA=SQRT(RSQM)
C     TALLY CPA RANGES INTO BOXES
C
      IF(RCPA.GE.59.0) GO TO 610
      LR=DX* RCPA +1.0
      NTALLY(LR)=NTALLY(LR)+ 1
      GO TO 1150
  610 NTALLY(60)=NTALLY(60) +1
C
C     INCREMENT NUMBER OF RUNS
C
 1150 NRUNS=NRUNS +1
C
C     TEST TO SEE IF DESIRED NUMBER OF RUNS OBTAINED
C
      IF(NRUNS .EQ. NTRUN) GO TO 1200
      GO TO 373
 1200 CONTINUE
C
C     RESET ITERATION COUNTER TO ZERO
C
      NRUNS=0
C
C     COMPUTE TOTAL NUMBER OF TALLIES
C
      NTSUM=0.0
      DO 650 L=1,60
      NTSUM= NTSUM + NTALLY(L)
  650 CONTINUE
```

```
C
C     COMPUTE INDIVIDUAL RANGE PROBABILITIES AND THE
C     CUMULATIVE PROBABILITIES
C
      CDF(1)=0.0
      SUMP=0.0
      DO 670 M=1,60
      MM=M+1
      PBOX=FLOAT(NTALLY(M))/FLOAT(NTSUM)
      SUMP=SUMP + PBOX
      CDF(MM)=SUMP
  670 CONTINUE
C
C
C     RESET ALL TALLY BOX VALUES TO ZERO
C
      DO 680 I=1,60
      NTALLY(I)=0
  680 CONTINUE
C
C
C     COMPUTE PARAMETERS FOR THE ALTERNATE SEARCH PLAN
C
C
      CMULT=2.0
      FACTOR=0.8
  685 FACTOR=0.8
  690 CONTINUE
      XDIST=FACTOR*STE*TL
C
C     COMPUTE ALTERNATE SEARCH LEG LENGTHS
C
      DO 700 MM=1,24
      CMULT=OMULT**MM
      SLO(MM)=CMULT*XDIST
  700 CONTINUE
C
C     CALCULATE X AND Y COORDINATES OF THE  SEARCH LEGS
C
      XCO(1)=CRXC-XDIST
      YCO(1)=CRYC
      XCO(2)=XCO(1)
      YCO(2)=YCO(1)+SLO(1)
      XCO(3)=XCO(2)+SLO(2)
      YCO(3)=YCO(2)
      XCO(4)=XCO(3)
      YCO(4)=YCO(3)-SLO(3)
      DO 710 NN=1,5
```

```
      NNO=4*(NN)
      NN1=4*(NN)+1
      NN2=4*(NN)+2
      NN3=4*(NN) +3
      NN4=4*(NN) +4
      XCO(NN1)=XCO(NNO)-SLC(NNO)
      YCO(NN1)=YCO(NNO)
      XCO(NN2)=XCO(NN1)
      YCO(NN2)=YCO(NN1)+ SLC(NN1)
      XCO(NN3)=XCO(NN2)+ SLC(NN2)
      YCO(NN3)=YCO(NN2)
      XCO(NN4)=XCO(NN3)
      YCO(NN4)=YCO(NN3)-YCO(NN3)
  710 CONTINUE
      XCO(25)=XCO(24)-SLC(24)
      YCO(25)=YCO(24)
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG ONE
C
      LV=1
  810 B1(LV)=XCO(LV)
      B2(LV)=0.0
      B3(LV)=YCO(LV)
      B4(LV)=SO
      LV=LV+4
      IF(LV.EQ.25) GO TO 815
      GO TO 810
  815 CONTINUE
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG TWO
C
      LV=2
  820 B1(LV)=XCO(LV)
      B2(LV)=SO
      B3(LV)=YCO(LV)
      B4(LV)=0.0
      LV=LV+4
      IF(LV.EQ.26) GO TO 825
      GO TO 820
  825 CONTINUE
C
C     DETERMINE INTIAL VALUES FOR SEARCH LEG THREE
C
      LV=3
  830 B1(LV)=XCO(LV)
      B2(LV)=0.0
      B3(LV)=YCO(LV)
      B4(LV)=-SO
```

```
      LV=LV+4
      IF(LV.EQ.27) GO TO 835
      GO TO 830
  835 CONTINUE
C
C     DETERMINE INITIAL VALUES FOR SEARCH LEG FOUR
C
      LV=4
  840 B1(LV)=XCO(LV)
      B2(LV)=-SO
      B3(LV)=YCO(LV)
      B4(LV)=0.0
      LV=LV+4
      IF(LV.EQ.28) GO TO 850
      GO TO 840
  850 CONTINUE
C
C     CALCULATION OF TIMES AT WHICH SEARCHER REACHES SEARCH
C                         LEG   COORDINATES
C
      TCO(1)=TL
      SLOSUM=TL
      DO 860 III=2,25
      NSLO=(III)-1
      SLOSUM=SLOSUM + SLO(NSLO)/SO
      TCO(III)=SLOSUM
      IF(TCO(III).GT. TLIMIT ) GO TO 870
  860 CONTINUE
  870 CONTINUE
C
C     SELECT  TGT COURSE UNIFORMLY BETWEEN UPPER AND LOWER LIMIT
C
  880 CONTINUE
      IC=IC*KR
      RN1=0.5 +FLOAT(IC) * 2.328306E-10
      CT=RN1*(ULCT-LLCT) +LLCT
C
C     GENERATE DIRECTION AND DISTANCE OF DATUM UNCERTAINITY
C
      IC=IC*KR
      RN2=0.5 +FLOAT(IC) *2.328306E-10
      DDL=RN2*2.0*3.14159
  890 Z=0.0
      DO 900 J=1,12
      IC=IC*KR
      Z=Z+ FLOAT(IC) *2.328306E-10
      RN3=SIGMA *Z +FLOAT(MU)
      DSDU= ABS(RN3)
```

```
  900 CONTINUE
C
C     GENERATE TARGET SPEED UNIFORMLY BETWEEN UPPER AND LOWER LIMITS
C
      IC=IC*KR
      RN4=0.5+FLOAT(IC)*2.328306E-10
      STA=RN4*(ULST-LLST)+LLST
C
C     CALCULATE TARGET X AND Y VELOCITY COMPONENTS
C
  910 VTX=STA* SIN(CT)
      VTY=STA*COS(CT)
C
C     CALCULATE INITIAL TARGET X AND Y COORDINATES
C
      TGTXC=ORXC + DSDU * COS(DDU)
      TGTYC=ORYC + DSDU * SIN (DDU)
C
C     DETERMINE CANDIDATES FOR MINIMUM FROM SEARCH LEG INTERVAL
C
      DO 1000 K=1,NSLO
      KK=(K)+1
      TIMERT=-(( B1(K)-TGTXC)*(B2(K)-VTX)+(B3(K)-TGTYC)*(B4(K)-VTY))/
     1((B2(K)-VTX)**2 +(B4(K)-VTY)**2)
      IF(TIMERT .LE.0.0) GO TO 1010
      IF(TIMERT.GT. TCO(KK)) GO TO 1010
      IF(TIMERT.LT.TCO(K)) GO TO 1010
      IF(TIMERT.GT. TLIMIT) GO TO 1010
      SXC=A1(K)+A2(K)*TIMERT
      SYC=A3(K)+A4(K)*TIMERT
      TXC=VTX* TIMERT +TGTXC
      TYC=VTY * TIMERT + TGTYC
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
      GO TO 1000
 1010 RSQ(K)=9.9999E+10
 1000 CONTINUE
C
C     DETERMINE END POINT CANDIDATES FOR MINIMUM
C
      KPLUS3=NSLO+1
      KPLUS4=2 * NSLO +1
      DO 1020  K=KPLUS3 , KPLUS4
      K1=(K)-NSLO
      TIME=TCO(K1)
      IF(TIME.GE.TLIMIT) GO TO 1030
      SXC=XCO(K1)
      SYC=YCO(K1)
      TXC=TGTXC +VTX*TIME
```

```
      TYC=TGTYC + VTY * TIME
      RSQ(K)=(SXC-TXC)**2 +(SYC-TYC)**2
 1020 CONTINUE
      GO TO 1040
 1030 KPLUS4=(K)-1
 1040 CONTINUE
C
C     DETERMINE THE CLOSEST POINT OF APPROACH FOR THIS ITERATION
C
      RSQM=RSQ(1)
      DO 1050 K=2, KPLUS4
      RMIN=AMIN1(RSQM,RSQ(K) )
      RSQM=RMIN
 1050 CONTINUE
      RCPA=SQRT(RSQM)
C
C     TALLY CPA RANGES INTO BOXES
C
      IF(RCPA.GE.59.0) GO TO 1060
      LR=DX* RCPA +1.0
      NTALLY(LR)=NTALLY(LR)+ 1
      GO TO 1250
 1060 NTALLY(60)=NTALLY(60)+1
C
C     INCREMENT NUMBER OF RUNS
C
 1250 NRUNS=NRUNS+1
C
C     TEST TO SEE IF DESIRED NUMBER OF RUNS OBTAINED
C
      IF(NRUNS .EQ. NTRUN) GO TO 1300
      GO TO 880
 1300 CONTINUE
C
C     RESET ITERATION COUNTER TO ZERO
C
      NRUNS=0
C
C     COMPUTE TOTAL NUMBER OF TALLIES
C
      NTSUM=0.0
      DO 1310 L=1,60
      NTSUM= NTSUM + NTALLY(L)
 1310 CONTINUE
C
C     COMPUTE INDIVIDUAL RANGE PROBABILITIES AND THE
C     CUMULATIVE PROBABILITIES
C
```

```
      CDF2(1)=0.0
      SUMP=0.0
      DO 1320 M=1,60
      MM=M+1
      PBOX=FLOAT(NTALLY(M))/FLOAT(NTSUM)
      SUMP=SUMP + PBOX
      CDF2(MM)=SUMP
 1320 CONTINUE
      WRITE(6,9710)
 9710 FORMAT('1')
      WRITE(6,9730)
 9730 FORMAT(//,10X,'SEARCH PLAN WAS CALCULATED USING THE FOLLOWING PARA
     1METERS')
      WRITE(6,9740) SO,STE,TL
 9740 FORMAT(20X,'SEARCHER SPEED=',F4.1,/20X,'ASSUMED TARGET SPEED=',F4.
     11,/20X,'TIME LATE=',F4.1)
      CALL PLOTP(X,CDF,61, 1)
      CALL PLOTP(X,CDF2,61,3)
      WRITE(6,9700)SO,STE,TL,TLIMIT
 9700 FORMAT('1',       //,25X,'SEARCHER SPEED=',F4.1,/,  25X,'ASSUMED TA
     2RGET SPEED=',F4.1,/,25X,'TIME LATE=',F4.1,/,25X,'TOTAL SEARCH TIME
     3=',F8.2,/ )
      WRITE(6,9805)
 9805 FORMAT(10X,'KOOPMAN',14X,'ALTERNATE',/,10X,'SEARCH',15X,'SEARCH',/
     2,10X,'PLAN',17X,'PLAN',/)
      WRITE(6,9806) FACTOR,CMULT
 9806 FORMAT(31X,'FACTOR=',F4.1,2X,'MULTIPLIER=',F6.2,/)
      DO 1330 N=1,61
      WRITE(6,9810) N,CDF(N),N,CDF2(N)
 9810 FORMAT(11X,'CDF',I3,'=',F6.4,8X,'CDF',I3,'=',F6.4)
 1330 CONTINUE
C
C     RESET ALL TALLY BOX VALUES TO ZERO
C
      DO 1340 I=1,60
      NTALLY(I)=0
 1340 CONTINUE
C
C     SUBROUTINE TO VARY THE X DISTANCE
C
      FACTOR=FACTOR+0.1
      IF(FACTOR.GE.1.5) GO TO 1350
      GO TO 690
C
C     SUBROUTINE TO VARY THE MULTIPLIER
C
 1350 CONTINUE
      CMULT=OMULT+ 0.5*OMULT
```

```
      IF(CMULT.GE. 8.0) GC TO 1360
      GC TO 685
 136C CCNTINUE
      STCP
      ENC
```

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Expanding Square Search
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Submarine Search

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This thesis investigates the effectiveness of a search plan developed by B. O. Koopman in a submarine versus submarine search situation. Two computer simulation models allow probability of target detection as a function of sonar range to be used as a measure of effectiveness. The Koopman search plan is analyzed and a family of alternate search plans are developed. The choice of a particular alternate search plan is

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dependent on the parameters of the problem. These parameters are target speed, searcher speed, time late to the search area and total time available to conduct the search. By use of the computer programs a search plan can be chosen so as to maximize the probability of target detection at a particular sonar range for each combination of input parameters.

Thesis 146273
S6054 Smith
c.1
Two evaluative models for a family of submarine versus submarine expanding square search plans.